Throughput-Optimal Flow Allocation on Multiple Disjoint Paths for Random Access Wireless Multi-hop Networks

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***Abstract*—In this paper we consider random access wireless multi-hop mesh networks with multi-packet reception capabilities where multiple flows are forwarded to the gateways through node disjoint paths. We address the issue of aggregate throughput optimal flow rate allocation with bounded packet delay guarantees for flows exhibiting both intra- and inter-path interference. We propose a distributed flow rate allocation scheme that formulates flow rate allocation as an optimization problem and show that the corresponding problem is non-convex using an illustrative topology. Furthermore we employ a simple model for the average aggregate throughput achieved by all flows that captures both intra- and inter-path interference. The proposed scheme is evaluated through NS-2 simulations of several random wireless scenarios. Simulation results reveal that our model accurately captures the average aggregate throughput achieved by all flows for all random scenarios employed. We also compare in terms of average aggregate throughput our flow allocation scheme with the following schemes: a scheme that assigns flows on paths on a round-robin fashion, one that optimally utilizes the best path only and another one that assigns the maximum possible flow on each path. Simulation results show that our scheme achieves significantly higher average aggregate throughput from these schemes.**

***Index Terms*—Multipath, flow allocation, random access.**

I. Introduction

In order to better utilize the scarce resources of wireless multi-hop networks and meet the increased user demand for QoS, numerous studies have suggested the use of multiple paths in parallel. Utilization of multiple paths can provide a wide range of benefits in terms of, throughput [2], delay [3], reliability [4], load balancing [5], [2], security [6] and energy efficiency [5], [7]. However multipath utilization in wireless networks is more complicated compared to their wired counterparts since transmissions across a link interfere with neighbouring links reducing thus network performance.

In this study we consider wireless random access multihop networks with multi-packet reception capabilities where multiple unicast flows are forwarded to their destinations through multiple multi-hop node disjoint paths. We focus on the issue of determining flow data rates that maximize the average aggregate network throughput in the presence of both intra- and inter-path interference for the aforementioned network settings. An additional goal apart from maximum aggregate throughput is bounded end-to-end packet delay.

The main contribution of this study is the design of a distributed flow rate allocation scheme that formulates flow rate allocation as an optimization problem. The key feature of the proposed scheme is that it maximizes the average aggregate throughput (AAT) achieved by all flows while also providing bounded packet delay guarantees. Another contribution of this study is a simple model for the average aggregate throughput, capturing both inter- and intra-path interference through the SINR model. As far as this model is concerned we also explore the trade-off between accurately capturing the AAT observed in the simulation scenarios and the complexity in formulating and solving the corresponding optimization problem. Interference among neighbouring links is approximated by considering only the *dominant* interferers for each link.

We motivate the problem of flow allocation and demonstrate the proposed scheme using a toy topology and show that the corresponding optimization problem is non-convex. We evaluate the proposed flow allocation scheme through NS-2 simulations applied to several random wireless scenarios. There is a slight deviation between the analytical results drawn from our model and the simulated ones concerning the average aggregate throughput achieved. The reason for this deviation is explained in section V. In the second part of the evaluation process we compare the AAT achieved by our scheme with the following flow allocation schemes: *Best-path* that optimally utilizes the best path available, *Maximum Flow Per Path* that assigns the maximum possible flow (one packet per slot) on each path and a *Round-Robin* based one. For all simulated scenarios explored and all SINR threshold values considered, our scheme achieves significantly higher throughput. Finally, as far as the aforementioned complexity-accuracy trade-off regarding ATT is concerned, we show that considering only a small number of interferers for each link results in a minor overestimation of the observed AAT in the simulated scenarios.

The rest of the paper is organized as follows: section II overviews the related work while section III presents the system model considered. In section IV we present how aggregate throughput optimal flow rate allocation is formulated as an optimization problem and demonstrate it through a toy

This work was presented in part in 9th IEEE Broadband Wireless Access Workshop [1].

M. Ploumidis was supported by "HERACLEITUS II - University of Crete", NSRF (ESPA) (2007-2013) and was co-funded by the European Union and national resources.

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topology. In section V we describe the simulation setup and present the evaluation results. We conclude this study in VI.

II. Related work

A wide range of different schemes have been proposed in literature focusing on multipath utilization for improving network performance, including routing schemes, resource allocation, flow control, opportunistic-based forwarding ones, e.t.c. A significant amount of studies focuses on identifying the set of paths that will guarantee improved performance in terms of some metric [2], [3], [8], [9]. However, such studies mostly address the issue of *which* paths should be utilized and rely on heuristic-based approaches concerning the issue of *how* should these paths be utilized (e.g. allocation of traffic on these paths) [2], [3], [4], [8]. In [8] for example, traffic is allocated on a round-robin fashion among the available paths.

Several studies focus on coordinating the access of multiple flows employing different paths to shared network resources suggesting joint scheduling, with routing, power control, or channel assignment. Authors in [10] for example suggest a scheme that performs joint channel assignment, scheduling and routing for maximizing system throughput. [11] proposes a resource allocation scheme for multiple flows in wireless networks that performs joint scheduling, routing and power control while the authors in [12] address the problem of joint routing, scheduling and power control for multiple information flows in interference limited ad hoc networks. The problem is formulated as a mixed-integer programming one and a polynomial time framework for solving it is suggested. Authors in [13] study an MPLS-based forwarding paradigm and aim at identifying a feasible routing solution for multiple flows deploying multiple paths. Links whose transmissions have a significant effect on each others success probability are considered to belong to the same collision domain and cannot be active at the same time.

[14] suggests a technique for combining multipath forwarding with packet aggregation over IEEE802.11 wireless mesh networks. Multipath utilization is accomplished by employing Layer-2.5, a multipath routing and forwarding strategy that aims at utilizing links in proportion to their available bandwidth.

As far as flow allocation on multiple paths and rate control is concerned, a well studied approach associates a utility function to each flow's rate and aims at maximizing the sum of these utilities subject to cross-layer constraints [15], [16]. Following [17], the Network Utility Maximization (NUM) framework has found many applications in rate control over wireless networks [18], [19]. Authors in [20], instead of employing a utility function of a flow's rate, they employ a utility function of flow's effective rate in order to take into account the effect of lossy links. Authors in [21] suggest a distributed rate allocation algorithm aimed at minimizing the total distortion of video streams transmitted over wireless adhoc networks. In [22] a max-min fair scheduling allocation algorithm is proposed along with a modified backoff algorithm for achieving long term fairness. In [23] a distributed flow control algorithm aimed at maximizing the total traffic flowing from sources to destinations also providing network lifetime guarantees is proposed. Based on the theoretical ideas of back-pressure scheduling and utility maximization, Horizon [24] constitutes a practical implementation of a multipath forwarding scheme that interacts with TCP.

There is also a significant amount of studies that suggest opportunistic forwarding/routing schemes that exploit the broadcast nature of the wireless medium. [25] suggests a multipath routing protocol called Multipath Code Casting that employs opportunistic forwarding combined with network coding. It also performs congestion control and employs a rate control mechanism that achieves fairness among different flows by maximizing an aggregate utility of these flows. Authors in [26] suggest an optimization framework that performs optimal flow control, routing, scheduling and rate adaptation employing multiple paths and opportunistic transmissions.

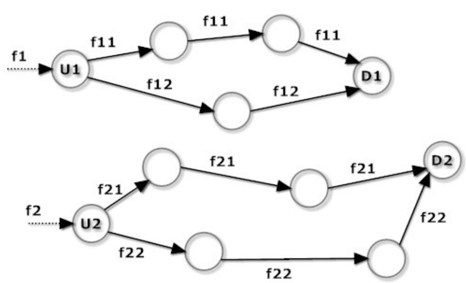
Different from all the above, in this study we address the issue of aggregate throughput optimal flow rate allocation for random access wireless multi-hop mesh networks with multi-packet reception capabilities where multiple flows are routed through node disjoint paths. Intra- and inter-path interference are captured through the SINR model. It should also be noted that the assumption of random access implies that transmitters get access to the shared medium in a decentralized manner without presupposing any coordination method. By maximizing the aggregate throughput of all flows instead of a sum over utility functions assigned to flow rates we take into account the intra- and inter-path interference imposed on neighboring links due to the flow injected on each path and thus their effect on aggregate throughput. Additionally in this way we can take into account the limit imposed on aggregate throughput by bottleneck links and also adjust the flow injected on each path based on the average throughput of its bottleneck link in order to prevent packets from accumulating at the queue of the sender of each bottleneck link.

III. System Model

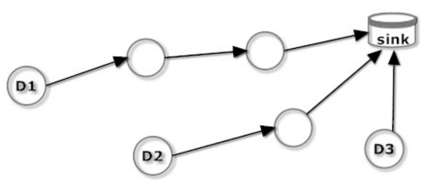
We consider static wireless multi-hop networks with the following properties:

- Random access to the shared medium where each node transmits independently of all other nodes requiring no coordination among them. When relay nodes are concerned, q_i is used to denote the packet transmission probability for node i given there is a packet available for transmission. For the relay nodes, q_i is assumed to be a parameter whose value is fixed and propagated periodically back to the sources through routing protocol's control messages. For flow originators it denotes the rate at which they inject packets into the network (flow rate).
- Time is slotted and each packet transmission requires one timeslot.
- Flows among different pairs of source and destination nodes carry unicast traffic of same-sized packets.
- All nodes use the same channel and rate, and are equipped with multi-user detectors being thus able to successfully decode packets from more than one transmitter at the same slot [27].

- We assume that all nodes are half-duplex and thus, cannot transmit and receive simultaneously.
- For the analysis we assume that all nodes have always packets available for transmission. However, in the simulation study we consider also the case that the nodes can have empty queues. As illustrated in Section V-B, there is no significant impact on the AAT.
- As far as routing is concerned, multiple disjoint paths are assumed to be available by the routing protocol, one for each flow. Moreover, source routing is assumed ensuring that packets of the same flow are routed to the destination along the same path. Apart from that, for each node its position, transmission probability or flow rate along with an indication of whether it is a flow originator are assumed known to all other nodes. This information can be periodically propagated throughout the network through a link-state routing protocol. Section V presents implementation details concerning these assumptions.



(a) Wireless multi-hop mesh network



(b) Sensor network

Fig. 1. Wireless scenarios where throughput optimal flow allocation is applicable

Figures 1(a) and 1(b) present two different wireless settings where the suggested flow allocation framework may be employed. In the first scenario depicted in figure 1(a) different users (U1 and U2) generate flows (f1, f2) that are routed to destination nodes D1 and D2 respectively through node disjoint multi-hop paths. These flows can be split into multiple subflows in order to aggregate network resources and achieve higher aggregate throughput. The suggested flow allocation framework can be applied in order to identify the data rates for subflows f11, f12, f21, and f22 that result in maximum average aggregate throughput for both users.

The second scenario depicted in figure 1(b) represents a sensor network where multiple sensor nodes generate data (D1, D2, and D3) that are forwarded to the sink through multiple paths comprised by relay sensor nodes. The proposed flow allocation scheme can be employed in order to maximize the rate at which the sink receives data from the sensor nodes.

A. Physical Layer Model

The MPR channel model used in this paper is a generalized form of the packet erasure model. In the wireless environment, a packet can be decoded correctly by the receiver if the received $SINR$ exceeds a certain threshold. More precisely, suppose that a given set T of nodes transmitting in the same time slot. Let $P_{rx}(i,j)$ be the signal power received from node i at node j. Let $SINR(i,j)$ be expressed using (1).

$$SINR(i,j) = \frac{P_{rx}(i,j)}{\eta_j + \sum_{k \in T \setminus \{i\}} P_{rx}(k,j)}. \qquad (1)$$

In the above equation η_j denotes the receiver noise power at j. We assume that a packet transmitted by i is successfully received by j if and only if $SINR(i,j) \geq \gamma_j$, where γ_j is a threshold characteristic of node j. The wireless channel is subject to fading; let $P_{tx}(i)$ be the transmitting power of node i and $r(i,j)$ be the distance between i and j. The power received by j when i transmits is $P_{rx}(i,j) = A(i,j)g(i,j)$ where $A(i,j)$ is a random variable representing channel fading. Under Rayleigh fading, it is known [28] that $A(i,j)$ is exponentially distributed. The received power factor $g(i,j)$ is given by $g(i,j) = P_{tx}(i)(r(i,j))^{-\alpha}$ where α is the path loss exponent with typical values between 2 and 4. The success probability of link (i,j) when the transmitting nodes are in T is given by

$$p^j_{i/T} = \exp\left(-\frac{\gamma_j \eta_j}{v(i,j)g(i,j)}\right) \prod_{k \in T \setminus \{i,j\}} \left(1 + \gamma_j \frac{v(k,j)g(k,j)}{v(i,j)g(i,j)}\right)^{-1}, \qquad (2)$$

where, $v(i,j)$ is the parameter of the Rayleigh random variable for fading. The analytical derivation for this success probability which captures the effect of interference on link (i,j) from transmissions of nodes in set T, can be found in [29].

IV. Analysis

In this section we present how aggregate throughput optimal flow rate allocation is formulated as an optimization problem for random topologies. The suggested scheme is demonstrated through a toy topology.

A. Throughput optimal flow rate allocation

The suggested method for formulating aggregate throughput optimal flow rate allocation as an optimization problem for random topologies is a procedure consisting of three steps. We demonstrate this procedure assuming multiple flows that are forwarded to the same destination. The same analysis however can be applied for the case where multiple flows have different destination nodes. First, the notations used in the analysis are presented and are also summarized in table I. V denotes the set of the nodes and $|V| = N$. We assume m flows $f_1, f_2, ..., f_m$, that need to forward traffic to the destination node D. $R = \{r_1, r_2, ..., r_m\}$ represents the set of m disjoint paths employed by these flows. $|r_i|$ is used to denote the number of links in path r_i. $I_{i,j}$ is the set of nodes that cause interference to packets sent from i to j. For example, if all network nodes are assumed to contribute with interference to link (i,j) and $j \neq D$,

TABLE I
NOTATIONS

Notation	Definition
V	Set of nodes. $\|V\| = N$
$f_1, f_2, ..., f_m$	m flows
r_i	Path i employed by flow f_i
$R = \{r_1, r_2, ..., r_m\}$	Set of node disjoint paths
$\|r_i\|$	Num of links in path r_i
$I_{i,j}$	Interfering nodes for link (i,j)
$I_{i,j}[n]$	Id of nth interfering node for link (i,j)
$L_{i,j} = \|I_{i,j}\|$	Number of nodes that interfere with transmissions on (i,j)
$Src(r_k)$	Source node of the k^{th} flow
$P_{r_k} = \prod_{(i,j)\in r_k} p^j_{i/i}$	End-to-end success probability for path r_k
$\bar{T}_{i,j}$	Average throughput for (i,j) (Pkts/slot)
$\bar{T}_{r_k}$	Average throughput for k^{th} flow (Pkts/slot)

then $I_{i,j} = V \setminus \{i, j, D\}$ and thus the set of nodes that cause interference to that link has size $L_{i,j} = |I_{i,j}| = |V| - 3$. Further on, $Src(r_k)$ is used to denote the source node of the k^{th} flow employing path r_k. $\bar{T}_{i,j}$ and $\bar{T}_{r_k}$ denote the average throughput measured in packets per slot achieved by link (i,j) and flow f_k forwarded over path r_k respectively. Finally, $I_{i,j}[n]$ denotes the id of the nth interfering node for link (i,j).

The first step of the suggested method consists of deriving the expression for the average throughput of a random link (i, j). Average throughput for that link, $\bar{T}_{i,j}$, can be expressed through (3).

$$\bar{T}_{i,j} = \sum_{l=0}^{2^{L_{i,j}}-1} P_{i,j,l} q_{i,j} \prod_{n=1}^{L_{i,j}} q^{b(l,n)}_{I_{i,j}[n]} (1 - q_{I_{i,j}[n]})^{1-b(l,n)}, \quad (3)$$

where,

$$q_{i,j} = \begin{cases} q_i & j = D \\ q_i(1 - q_j) & j \neq D \end{cases},$$

$P_{i,j,l} = p^j_{i/i \cup \{I_{i,j}[n],\ \forall\ n:\ b(l,n) \neq 0)\}}$,
$b(l, n) = l\ \&\ 2^{n-1}$, & is the logical bitwise AND operator.

The outcome of a transmission along link (i,j) during a slot depends on the amount of received interference. The interference depends on the set of other transmitters that are active during the same slot. A node i is active during a slot with probability q_i. For flow originators q_i denotes flow rate. As also described in section V-A, transmission probability and position for every node are propagated periodically to all other nodes through routing protocol's topology control messages. Position information is used to infer each link's success probability based on equation 2. As a flow's data rate is increased, the interference imposed on other links is also increased. Estimating thus a link's (i,j) throughput requires enumerating all possible subsets of active transmitters. Assuming the maximum number of interfering nodes and a network with N nodes, all such subsets of interfering nodes for (i,j) are $2^{L_{i,j}}$. For large networks enumerating all subsets of active transmitters may be computationally intractable. In section V we explore a variant of the suggested flow allocation scheme where only the k dominant interferers are taken into account for expressing the throughput of link (i,j). As also discussed in that section, dominant interferers for that link are considered those that impose the most significant amount of interference to packets received by j. In (3), l enumerates all possible subsets of active transmitters while $b(l, n)$ becomes one if the n^{th} node in $I_{i,j}$ is assumed active in the l^{th} subset examined.

The average aggregate throughput achieved by all flows is expressed through $\bar{T}_{aggr} = \sum_{k=1}^{m} \bar{T}_{r_k}$ where $\bar{T}_{r_k} = \min\limits_{(i,j)\in r_k} \bar{T}_{i,j}$. The second step of the suggested method consists of maximizing the average aggregate throughput while also guaranteeing bounded packet delay which results in non-smooth optimization problem P1:

$$\underset{S}{\text{Maximize}} \sum_{k=1}^{m} \min_{(i,j)\in r_k} \bar{T}_{i,j} \qquad (P1)$$

s.t:

$$\begin{aligned}
&(S1): \ 0 \leq q_{Src(r_k)} \leq 1,\ k = 1, ..., m \\
&(S2): \ \bar{T}_{Src(r_k),i} \leq \bar{T}_{j,l}, \\
&\qquad \{\forall i, j, k, l : (Src(r_k), i),\ (j, l) \in r_k, |r_k| > 1 \\
&\qquad k = 1, ..., m\},
\end{aligned}$$

where, $S = \{q_{Src(r_k)},\ \ k = 1, ..., m\}$. Constraint set S1 ensures that the maximum data rate for any flow does not exceed one packet per slot while also allowing paths that are not optimal to use, to remain unutilized. Constraint S2 ensures that the flow injected on each path, that is the throughput of that path's first link, is limited by the flow that can be serviced by any subsequent link of that path. In this way data packets are prevented from accumulating at the relay nodes guaranteeing thus bounded packet delay. For the rest of the paper this constraint will be referred to as *bounded delay constraint.*

P1 can be transformed to the following smooth optimization problem:

$$\underset{S'}{\text{Maximize}} \sum_{k=1}^{m} \begin{cases} \bar{T}_{Src(r_k),D}, & |r_k| = 1 \\ q'_{Src(r_k)}, & |r_k| > 1 \end{cases} \qquad (P2)$$

$s.t.:$

$$\begin{aligned}
&(S1): \ 0 \leq q_{Src(r_k)} \leq 1,\ k = 1, ..., m \\
&(S2): \ \bar{T}_{Src(r_k),i} \leq \bar{T}_{j,l}, \\
&\qquad \{\forall i, j, k, l : (Src(r_k), i),\ (j, l) \in r_k, |r_k| > 1 \\
&\qquad k = 1, ..., m\} \\
&(S3): \ 0 \leq q'_{Src(r_k)} \leq 1,\ \{\forall k : |r_k| > 1\} \\
&(S4): \ q'_{Src(r_k)} \leq \bar{T}_{i,j},\ \{\forall i, j, k : |r_k| > 1,\ (i, j) \in r_k\}
\end{aligned}$$

where, $S' = \{q_{Src(r_k)},\ k = 1, ..., m\} \cup \{q'_{src(r_k)} :\ |r_k| > 1\}$. For the rest of the paper we will refer to optimization problem P2 above as the *flow allocation optimization problem.*

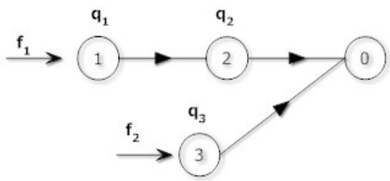


Fig. 2. Illustrative topology.

B. Throughput optimal flow rate allocation: Illustrative scenario

We consider the toy topology presented in Fig. 2. Two flows namely, f_1 and f_2, originating from nodes 1, and 3 are forwarded to destination node 0 through paths r_1: 1-2-0 and r_2: 3-0 respectively. We further assume that transmissions on a specific link cause interference to all other links. Before presenting each link's average throughput consider link $(2,0)$ as an example. Transmitters that cause interference to packets sent from 2 to 0 constitute set $I_{2,0} = \{1,3\}$ and thus $L_{2,0} = 2$. There are four possible subsets of nodes that may cause interference on link $(2,0) : \{\o\}, \{1\}, \{3\}, \{1,3\}$. When $l = 3$ in (3), it indicates the fourth subset of interfering nodes with $b(l,n)$ becoming one for both $n = 1$ and $n = 2$.

The average throughput per link is presented in (4a)-(4c).

$$\bar{T}_{1,2} = q_1(1-q_2)(1-q_3)p^2_{1/1} + q_1(1-q_2)q_3p^2_{1/1,3} \quad (4a)$$
$$\bar{T}_{2,0} = q_2(1-q_1)(1-q_3)p^0_{2/2} + q_2q_1(1-q_3)p^0_{2/2,1} + q_2(1-q_1)q_3p^0_{2/2,3} + q_2q_1q_3p^0_{2/1,2,3} \quad (4b)$$
$$\bar{T}_{3,0} = q_3(1-q_1)(1-q_2)p^0_{3/3} + q_3q_1(1-q_2)p^0_{3/1,3} + q_3(1-q_1)q_2p^0_{3/2,3} + q_3q_1q_2p^0_{3/1,2,3} \quad (4c)$$

Recall that q_1 and q_3 denote the data rates for flows f_1 and f_2 respectively. Aggregate average throughput achieved by all flows can be expressed through (5).

$$\bar{T}_{aggr} = \bar{T}_{r_1} + \bar{T}_{r_2}, \quad where, \quad \bar{T}_{r_1} = min\{\bar{T}_{1,2}, \bar{T}_{2,0}\}, \quad \bar{T}_{r_2} = \bar{T}_{3,0} \quad (5)$$

Aggregate throughput optimal flow rate allocation consists of identifying rates q_1, q_3 that maximize average aggregate throughput while also guaranteeing bounded packet delay. These rates can be found by solving the following optimization problem:

$$\begin{array}{lll} \underset{q_1,q_3}{\text{Maximize}} & \bar{T}_{30} + min\{\bar{T}_{12}, \bar{T}_{20}\} & \\ \text{subject to} & 0 \le q_i \le 1, \ i \in \{1,3\} & (g1)-(g4) \\ & \bar{T}_{12} \le \bar{T}_{20} & (g5) \end{array}$$

Constraint (g5) constitutes the bounded delay constraint for path r_1. According to third step of the process presented in the previous subsection, the above non-smooth optimization problem can be transformed to the following smooth optimization problem:

$$\begin{array}{lll} \underset{q_1^{'},q_1,q_3}{\text{Maximize}} & \bar{T}_{30} + q_1^{'} & \\ \text{subject to} & 0 \le q_i \le 1, \ i \in \{1,3\} & (g1)-(g4) \\ & \bar{T}_{12} \le \bar{T}_{20}, & (g5) \\ & q_1^{'} \le \bar{T}_{12}, & (g6) \\ & q_1^{'} \le \bar{T}_{20}, & (g7) \\ & 0 \le q_1^{'} \le 1 & (g8)-(g9) \end{array} \quad (P3)$$

Transforming the above optimization problem in the standard from, the cost function along with $g_5(q_1^{'}, q_1, q_3) = \bar{T}_{12} - \bar{T}_{20}$, $g_6(q_1^{'}, q_1, q_3) = q_1^{'} - \bar{T}_{12}$, $g_7(q_1^{'}, q_1, q_3) = q_1^{'} - \bar{T}_{20}$ related to constraints $g5$, $g6$, and $g7$ respectively are non-convex, thus the problem is non-convex.

Before presenting simulation results for random wireless topologies, we further motivate flow rate allocation on multiple paths using analytical results derived from the toy topology depicted in Fig 2. Let $d(i,j)$ denote the distance between nodes i and j. Let also $P_{r_k} = \prod_{(i,j)\in r_k} p^j_{i/i}$ denote the *end-to-end success probability* for path r_k. For the illustrative purpose of this section we assume that $d(1,2) = d(2,0) = d(3,1) = d$, $d(3,0) = \sqrt{5}d$, $d(3,2) = \sqrt{2}d$, where $d = 400m$. Further on, the path loss exponent assumed is 3 while transmission probability for relay node 2 is 0.5. Flow rates q_1 and q_3 that achieve maximum average aggregate throughput (AAT) for SINR threshold values $\gamma = \{0.25, 0.5, ..., 2\}$ are estimated by solving the optimization problem (P3) using the simulated annealing method. It should be noted that multi-hop path r_1 : 1-2-0 exhibits higher end-to-end success probability than path r_2 : 3-0 for all values of γ considered.

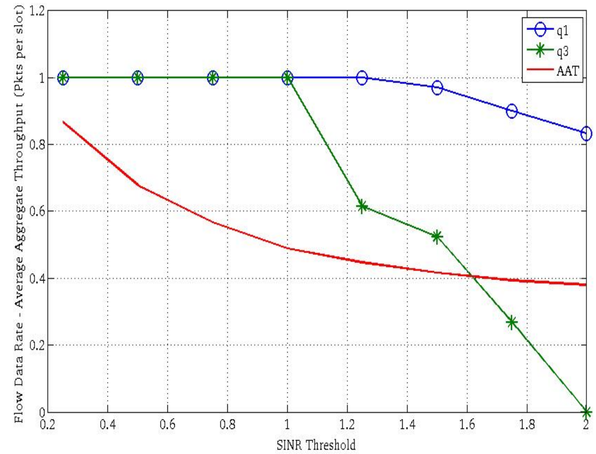


Fig. 3. Optimal Flow rates and Average Aggregate Throughput achieved.

In Fig. 3 we present throughput optimal flow rates assigned on paths r_1, and r_2 along with the average aggregate throughput achieved (AAT) for the aforementioned γ values. As this figure shows, the maximum AAT is achieved by full rate utilization of both paths for SINR threshold values up to 1.0 suggesting that inter-flow interference is balanced by the gain in throughput. For SINR threshold values larger than

TABLE II
PARAMETERS USED IN THE SIMULATIONS

Parameter	Value
Max Retransmit Threshold	3
Transmit Power	0.1 Watt
Noise power	7×10^{-11} Watt
Contention Window	5
Packet size	1500 Bytes
Path loss exponent	4

1.0, utilization of path r_2 which exhibits lower performance in terms of end-to-end success probability declines. This is due to the fact that for large SINR threshold values the effect of interference imposed on path r_1 becomes more significant. At the same time, flow forwarded through path r_2 manages to deliver only a small portion of its traffic to destination node 0.

V. EVALUATION

A. Simulation setup

We evaluate the proposed aggregate Throughput Optimal Flow Rate Allocation scheme (referred to as TOFRA for the rest of the paper) using network simulator NS-2, version 2.34 [30].

Concerning medium access control, a slotted aloha-based MAC layer is implemented. Transmission of data, routing protocol control and ARP packets is performed at the beginning of each slot without performing carrier sensing prior to transmitting. Acknowledgements for data packets are sent immediately after successful packet reception while failed packets are retransmitted. Slot length, T_{slot}, is expressed through: $T_{slot} = T_{data} + T_{ack} + 2D_{prop}$ where T_{data} and T_{ack} denote the transmission times for data packets and acknowledgements (ACKs) while D_{prop} denotes the propagation delay. It should be noted that all packets have the same size shown in table II. All network nodes, apart from sources of traffic, select a random number of slots before transmitting drawn uniformly from $[0, CW]$. The contention window (CW) is fixed for the whole duration of the simulation and equal to 5.

As far as physical layer is concerned, all data packets are successfully decoded if their received SINR exceeds the SINR threshold. The received SINR for each packet is computed through equation (1) and the path loss exponent is assumed to be $\alpha = 4$. Transmitters during each slot, that are considered to cause interference, are those transmitting data packets or routing protocol control packets. All nodes use the same SINR threshold, transmission rate and channel. Transmission power and noise is 0.1 Watt and 7×10^{-11} Watt respectively.

As far as routing is concerned we implement a multipath, source-routed link-sate routing protocol based on UM-OLSR [31]. Hello and Topology Control (TC) messages are periodically propagated throughout the network. Each topology control message carries the following information: a)transmission probability b)position, and c)an indication of whether it is a flow originator or not. As also discussed in section III, transmission probabilities are assumed to be fixed for relay nodes since contention window (CW) remains fixed for the whole simulation period. For flow originators transmission probabilities are estimated by solving the corresponding version the flow allocation optimization problem (presented in section IV) using the simulated annealing method. Using this information from the TC messages each node can infer both the network topology and the success probability for each link based on equation 2 since all link distances are known. Multipath set population is performed at flow sources each time a new TC message is received. As already stated, the main focus of the study is not only *which* paths should be employed to achieve improved performance but also *how* should these paths be utilized in order to maximize the average aggregate throughput (AAT) while also providing bounded packet delay guarantees. We thus employ a simple algorithm that provides traffic sources with multiple, link-disjoint, least-cost paths. The multipath set is populated on an iterative manner. On each iteration a specific flow's source and destination node are considered. The graph inferred from TC messages is searched for a least cost path using the Dijkstra algorithm. The nodes participating in the path identified are removed and the search process continues with the next flow's source and destination node. In this way, the multipath set consists of node disjoint paths.

Upon each TC message reception, each flow source performs the following two tasks as part of the proposed flow allocation scheme: first it applies the aforementioned algorithm for populating the multipath set. Then it solves the topology-specific instance of the flow allocation optimization problem presented in Section IV using the simulated annealing method. In this way the flow rates (packets per slot) that should be assigned on each path in order to achieve maximum aggregate throughput are estimated along with the average aggregate throughput for all flows. According to this process, flow rates are estimated on a distributed manner for all flow originators. Traffic sources generate UDP unicast flows and are kept backlogged for the whole simulation period.

As far as queues at the relay nodes are concerned, two variants are simulated for each forwarding scheme explored. The first variant follows the assumption of saturated queues in the analysis while in the second variant queues are not assumed to be saturated. The goal of this process is to gradually evaluate whether the suggested flow allocation scheme accurately captures the average aggregate throughput achieved. With the first variant we explore whether the suggested model for the AAT accurately captures the effect of random access and interference on AAT. The second variant explores the effect of the assumption concerning saturated queues on accurately capturing the AAT observed in the simulated scenarios. In order to implement the first variant the following patch is required in the routing module in Ns2: whenever a relay node *i* successfully receives a packet destined for a next hop *j*, it buffers the full header of the packet. Then, if the queue for the next hop gets empty during a subsequent slot, it creates a new packet with a dummy payload and adds the header buffered. In this way there is always a packet in the queue of *i* destined for *j*. The only exception to this is the interval required for

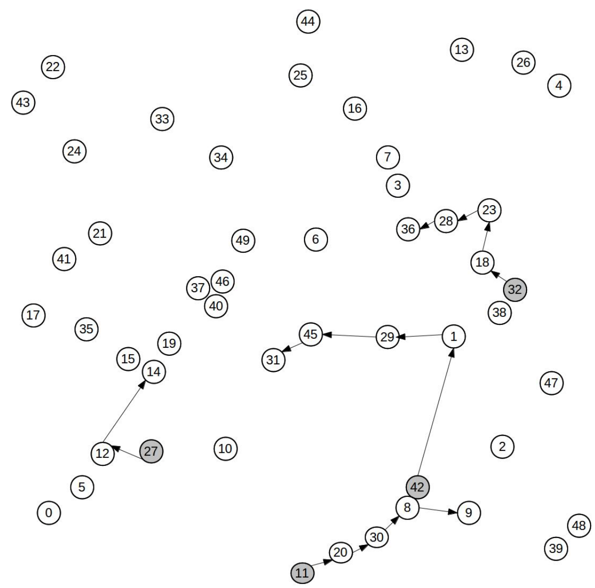


Fig. 4. Illustrative random wireless scenario

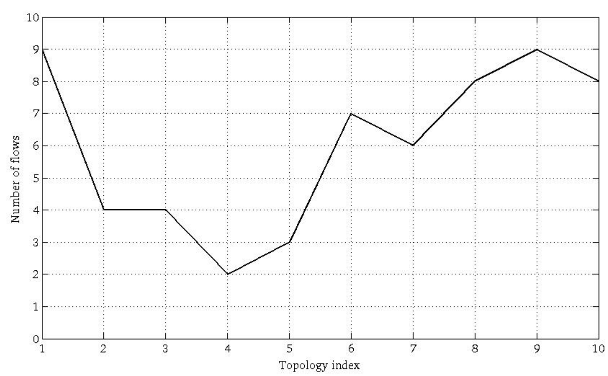


Fig. 5. Number of flows per wireless scenario

a successful packet reception at *j*. Until the first successful data packet reception, a relay node may have an empty data queue. This is one of the reasons for the deviation in terms of AAT between the analytical and simulation results since the possibility of some queues being empty suggests that the specific transmitter will have no packet to transmit resulting in lower network interference and thus to higher AAT.

For the purposes of the evaluation process presented in the next subsection, ten different wireless scenarios are generated. In each scenario, 50 nodes are uniformly distributed over an area of 500mx500m. The number of flows generated along with the source and destination node for each flow are selected randomly. A maximum number of ten flows is allowed for each scenario and the simulation time is 60 seconds.

Figure 5 shows the number of flows generated for each one of the ten wireless scenarios employed. Figure 4 depicts one such random wireless scenario including four flows. The source and destination nodes for flow f_1 are 42 and 31 respectively. The corresponding source and destination nodes for flows f_2, f_3, and f_4 are: $(11, 9)$, $(32, 36)$, and $(27, 14)$. When the suggested flow allocation scheme is employed for this scenario, it first searches for a multipath set for these flows as described above. As also shown in this figure the paths employed for these flows are: $r_1 : 42-1-29-45-31$, $r_2 : 11-20-30-8-9$, $r_3 : 32-18-23-28-36$, and $r_4 : 27-12-14$. Then it searches for flow rates q_{42}, q_{11}, q_{32}, and q_{27} that will provide with the highest average aggregate throughput while also guaranteeing bounded packet delay. In order to capture the effect of interference on success probability and thus on throughput different γ values are considered. The corresponding values are $0.5, 1.0, 1.5$, and 2.0 respectively.

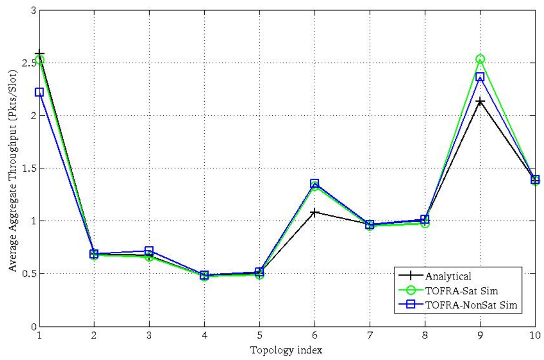
B. Simulation Results

The evaluation process consists of three parts. In the first one we explore whether the model employed by the proposed flow allocation scheme accurately captures the average aggregate throughput (AAT) achieved by all flows in the simulated scenarios. To introduce the notation used in the figures below, the case of the suggested flow allocation scheme that is simulated assuming saturated queues is denoted by *TOFRA-Sat Sim* while the second one where queues are not saturated is *TOFRA-NonSat Sim*.

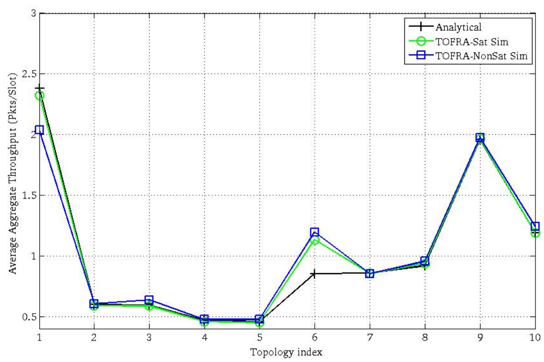
Figures 6(a) to 6(d) compare analytical with simulation results concerning average aggregate throughput (AAT) for SINR threshold values 0.5, 1.0, 1.5, and 2.0 and the ten different wireless scenarios explored. As these figures show, when queues are assumed saturated, the model employed by our flow allocation framework accurately captures the AAT achieved for all values of γ in all wireless scenarios. In some scenarios our model slightly underestimates the AAT observed in the simulation results. As already explained in the previous section this is due to how saturated queues are implemented. Queues at the relay nodes may remain with no packet to transmit until the first successful packet reception. This results in lower interference imposed on neighbouring links due to absence of transmissions.

In some scenarios also, our model overestimates the AAT observed in the simulation results. This is due to the fact that in our analysis we disregard control traffic due to routing protocol and ARP protocol and consider that all slots carry data packets. Additionally, in our analysis, we consider interference caused only by the nodes that participate in the paths employed. However, the nodes that do not participate in the multipath employed, periodically transmit control traffic and thus cause interference.

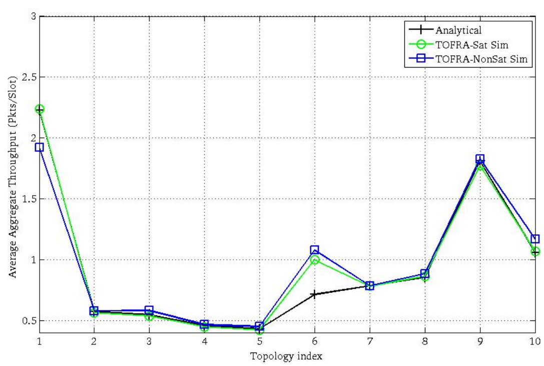
The average deviation over all simulated scenarios between the analytical and simulation results is 3.3%, 2.5%, 2.9%, and 3.5% respectively for the four SINR threshold values considered suggesting that the the simplifying assumptions adopted in our analysis have an insignificant effect on accurately capturing AAT. Figures 6(a) to 6(d) also show that when the assumption of saturated queues is removed from the simulation setup, there is an insignificant deviation between the average aggregate throughput achieved by the two variants of TOFRA. More precisely, collating the analytical results with



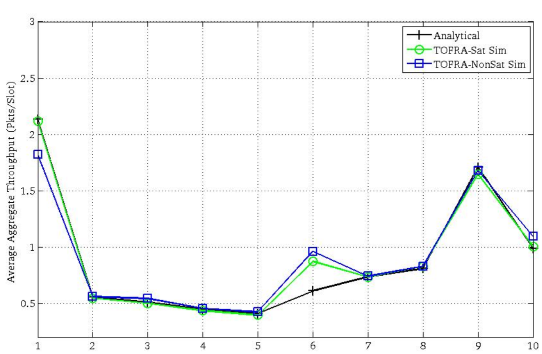
(a) γ=0.5



(b) γ=1.0



(c) γ=1.5



(d) γ=2.0

Fig. 6. Average Aggregate Throughput: Analytical vs. Simulation results.

the simulated ones when queues are not saturated shows that the average deviation over all simulated scenarios between the analytical and simulated results is 4.2%, 4.9%, 5.5%, and 5.7% respectively for $\gamma = 0.5, 1.0, 1.5, 2.0$.[1]

As far as AAT is concerned, for the topology presented in figure 4 and the TOFRA variant where queues are not assumed saturated, the variance of the aggregate throughput is 4×10^{-4}, 3×10^{-4}, 2×10^{-4}, and 1×10^{-4} for the four γ values considered.

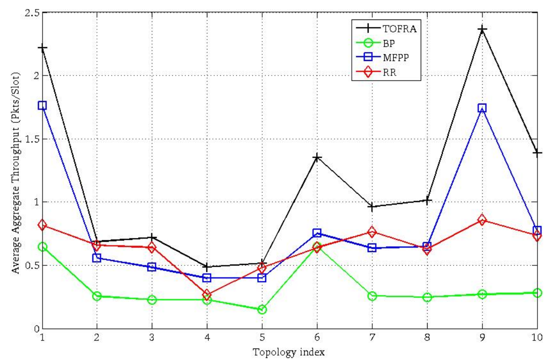
In the second part we compare our scheme (TOFRA) with three other flow allocation schemes, namely, *Best-Path (BP)*, *Maximum Flow Per Path (MFPP)*, and *Round-Robin (RR)*. Best-path employs a single path to the destination which is the one that exhibits the highest end-to-end success probability (defined in Section IV-B) and estimates the flow that should be assigned on this path by solving a single path version of the flow allocation optimization problem. MFPP assigns a flow rate of one packet per slot on each path while, round-robin employs a different path each time slot. For the evaluation process we consider the simulated variant for each scheme where the assumption of saturated queues is removed.

Figures 7(a)-7(d), collate the AAT achieved by all aforementioned schemes for the ten random scenarios employed. Each figure corresponds to one of the different SINR threshold values considered (0.5, 1.0, 1.5, and 2.0). As these figures show, TOFRA achieves significantly higher throughput than all other schemes in all the scenarios employed. More precisely TOFRA achieves 204.0%, 180.0%, 168.5%, and 156.2% higher AAT than BP on average for $\gamma = 0.5, 1.0, 1.5, 2.0$ respectively. The main reason for this gain over BP is that the proposed scheme manages to aggregate network resources of different paths in an intra- and inter-path interference-aware manner. Our scheme also achieves higher AAT than MFPP. The main reason for this is that it takes into account the effect of both intra- and inter-path interference on throughput. MFPP on the other hand assigns the maximum flow data rate on each path (one packet per slot) disregarding the effect of interference. TOFRA achieves 32.4%, 41.3%, 51.3%, and 64.2% higher average aggregate throughput on average than MFPP for $\gamma =$ 0.5, 1.0, 1.5, 2.0 respectively. Finally the average improvement in terms of ATT of the proposed scheme over RR is 52.8%, 48.1%, 45.2%, and 42.8% for the aforementioned γ values.

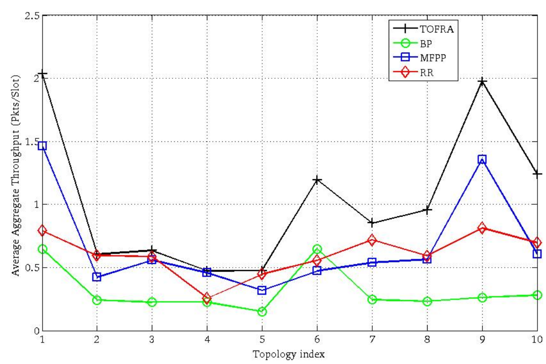
In the last part of the evaluation process we relax the way in which interference is captured by our model. The goal is to reduce the complexity of expressing the average aggregate throughput of all flows and consequently of solving the flow allocation optimization problem. As already described in Section IV, the first step for the process of formulating flow allocation as an optimization problem is deriving the expression for a random link's throughput. Relaxing the way in which interference relations are captured takes place in this part of the process.

Instead of considering all possible interfering nodes for expressing the average throughput achieved over that link, we approximate the interference imposed on it by taking into

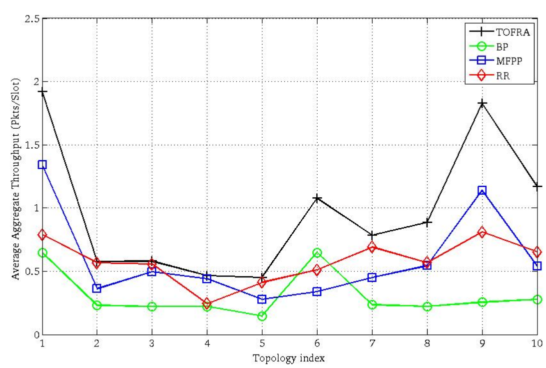
[1]This gap is explained because when a node has an empty queue in a timeslot, then it will remain silent. Thus, it will not cause interference to the other links.



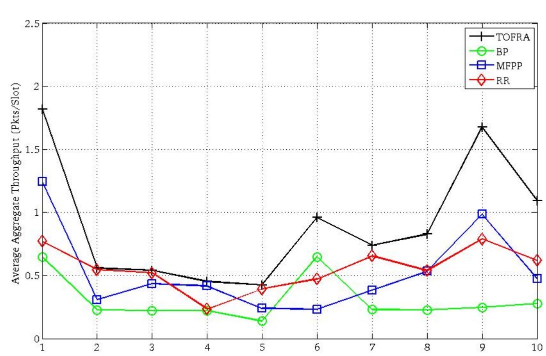
(a) γ=0.5



(b) γ=1.0

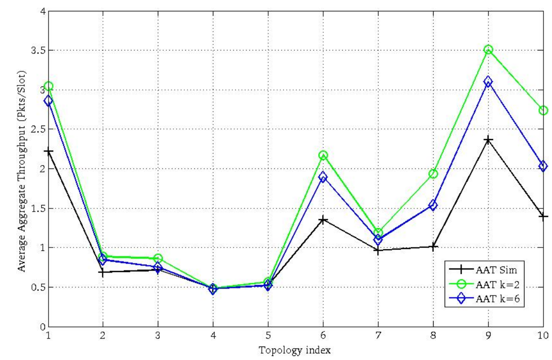


(c) γ=1.5

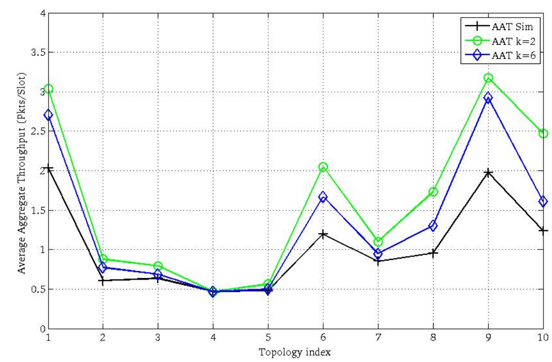


(d) γ=2.0

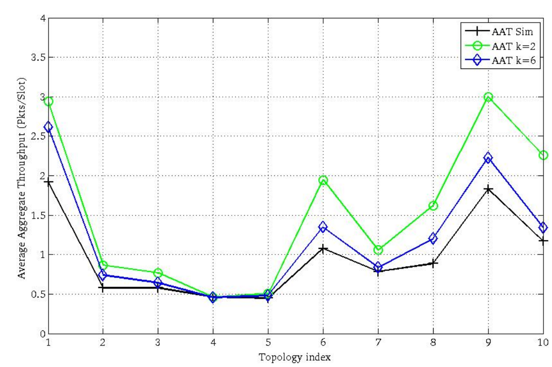
Fig. 7. Simulation results: AAT for TOFRA, MFPP, BP, and RR



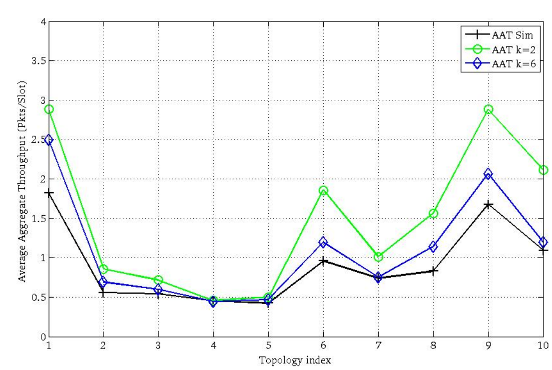
(a) γ=0.5



(b) γ=1.0



(c) γ=1.5



(d) γ=2.0

Fig. 8. Effect of number of dominant interferers on average aggregate throughput accuracy.

account only the *k dominant interferers*. The term dominant interferers refers to transmitters that contribute with the most significant amount of interference to packet receptions over a specific link and thus have the most significant effect on its success probability. The suggested approximation is expected to result in an AAT estimation that will be higher than the one observed in the simulated scenarios.

The purpose of this part of the evaluation process is to explore the trade-off between reduced complexity in formulating flow allocation as an optimization problem and accuracy in capturing the average aggregate throughput observed in the simulated scenarios. For each pair of wireless scenario and γ value (0.5, 1.0, 1,5, and 2.0) the corresponding optimization problem is formulated and solved considering a different number of dominant interferers (k=2,...,6).

Figures 8(a)-8(d) compare the AAT observed in the simulation results, denoted as AAT-sim, with the one estimated by our flow allocation scheme for the different values of dominant interferers considered. In order to make the corresponding plots more readable, analytical values concerning AAT for $k = 2$, and 6 are only included. Analytical results for a low number of dominant interferers are presented to demonstrate the gain in terms of accuracy achieved by slightly increasing the number of dominant interferers considered. As these plots show, taking into account the six dominant interferers results in an AAT estimation by the suggested model that is slightly higher than the one observed in the simulation results. To be more precise, considering only the six dominant interferers results in an estimation of ATT that is 17.07%, $16,8\%$, 13.2%, and 12.6% higher on average over all scenarios than the one observed in the simulated results for $\gamma = 0.5, 1.0, 1.5$, and 2.0 respectively.

We expect however that the number of dominant interferers that are sufficient to accurately capture each link's throughput will depend both on the density of the topology and the number of neighbouring links activated in order to forward data packets for flows. Figures 8(a)-8(d) for example show that the scenarios where the suggested model proves more accurate in capturing the AAT achieved by all flows, despite the interference approximation employed are those with indexes $2-5$. Figure 5 also shows, these four scenarios are the ones with the lowest number of flows among all. Fewer flows means that fewer links are employed to forward data packets and thus fewer active neighbouring transmitters for each link.

VI. Conclusion

This study explores the issue of aggregate throughput optimal flow rate allocation for wireless multi-hop random access networks with multi-packet reception capabilities. Flows are forwarded over multiple disjoint interfering paths. We propose a distributed scheme that formulates flow rate allocation as an optimization problem aiming at maximizing the average aggregate throughput of all flows while also providing bounded packet delay guarantees. They key feature of the suggested scheme is that it employs a simple model for the average aggregate throughput achieved by all flows that accounts for both intra- and inter-path interference with interference being captured through the SINR model. A toy topology is employed to demonstrate the proposed scheme and also show that the corresponding optimization problem is non-convex. We evaluate the suggested flow allocation scheme using Ns-2 simulations of ten random wireless scenarios. Collating analytical with simulations results reveals that the suggested scheme accurately captures average aggregate throughput despite the simplifying assumptions adopted by our analysis. Moreover, it achieves significantly higher average aggregate throughput than best-path, maximum flow per path and a round-robin based flow allocation scheme. As part of the evaluation process we also explore the trade-off between reduced complexity in formulating flow allocation as an optimization problem and the accuracy in estimating the average aggregate throughput observed in the simulation results. Simulation results show that considering only a small number of the dominant interfering nodes is sufficient in order to capture the average aggregate throughput achieved by all flows without a significant loss in accuracy.

Part of our future work is to address fairness issues too apart from maximizing the aggregate throughput achieved by all flows. We also plan to consider multiple transmission rates and relax the assumption of fixed transmission probability by allowing a variable contention window for each relay node. In the present study we treat interference as noise. In future steps we aim at adopting more sophisticated approaches for interference handling, such as, successive interference cancellation and joint decoding [28]. Finally we aim at exploring the performance of the suggested flow rate allocation scheme under the assumption of bursty packet losses.

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