Steerable and scale-discretized wavelet analyses of the cosmic microwave background

Yves Wiaux

ADA V Conference

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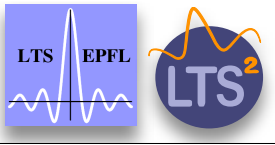


Introduction

- The cosmic microwave background - CMB
  - Precision laboratory for cosmology
- Scale-space analysis
  - Wavelets beyond spectral analysis
- Presentation overview

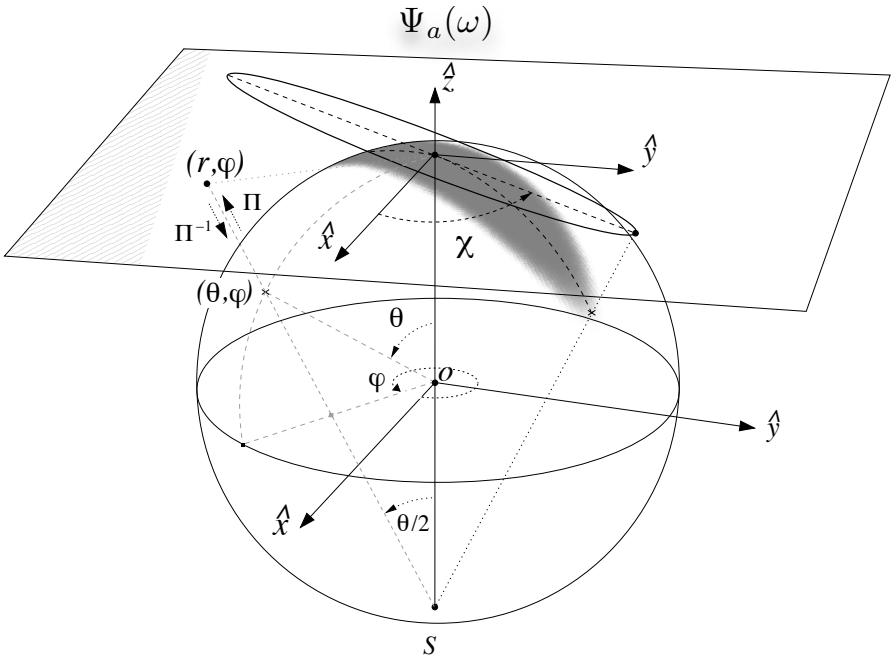
I. Steerable wavelets on the sphere
II. Scale-space CMB analyses
III. Scale-discretized wavelets on the sphere
IV. Non-Gaussian CMB component denoising

.I.

Steerable wavelets on the sphere

Continuous wavelets

Signals are analyzed by local filters which can be translated, rotated, and... **stereographically dilated with a continuous dilation factor**



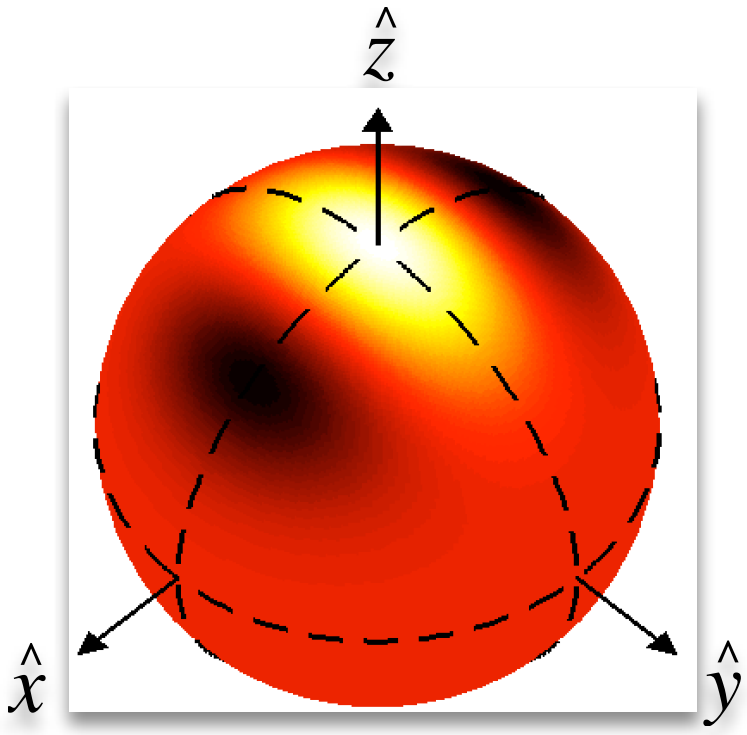
Wavelet coefficients: $W_\Psi^F(\rho, a) = \langle \Psi_{\rho,a} | F \rangle$

+ a correspondence principle with wavelets on the plane

- *ApJ 632 (2005) 15*
- *ApJ 652 (2006) 820*

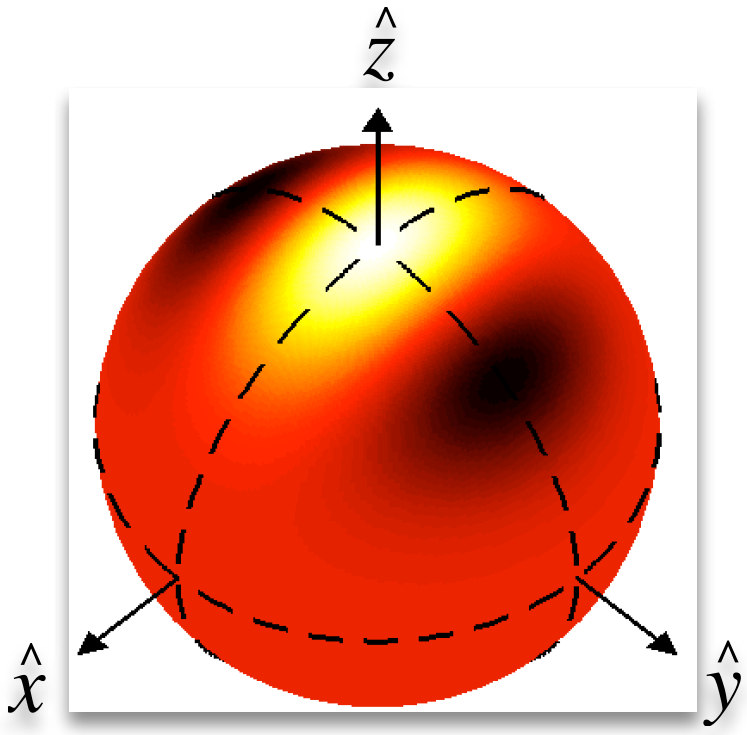
Steerability

Second gaussian derivative (2GD): basis wavelet 1



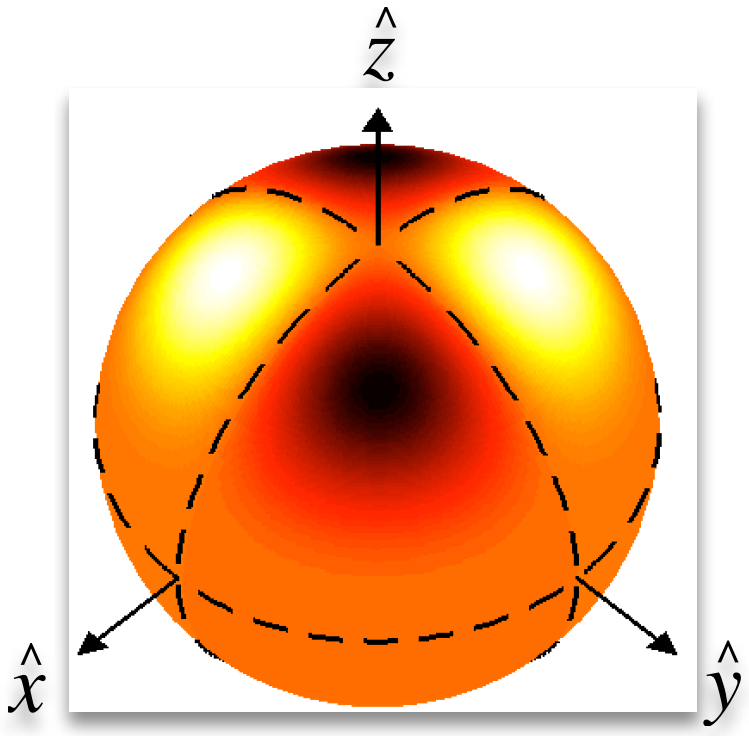
Steerability

Second gaussian derivative (2GD): basis wavelet 2

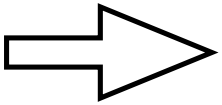


Steerability

Second gaussian derivative (2GD): basis wavelet 3

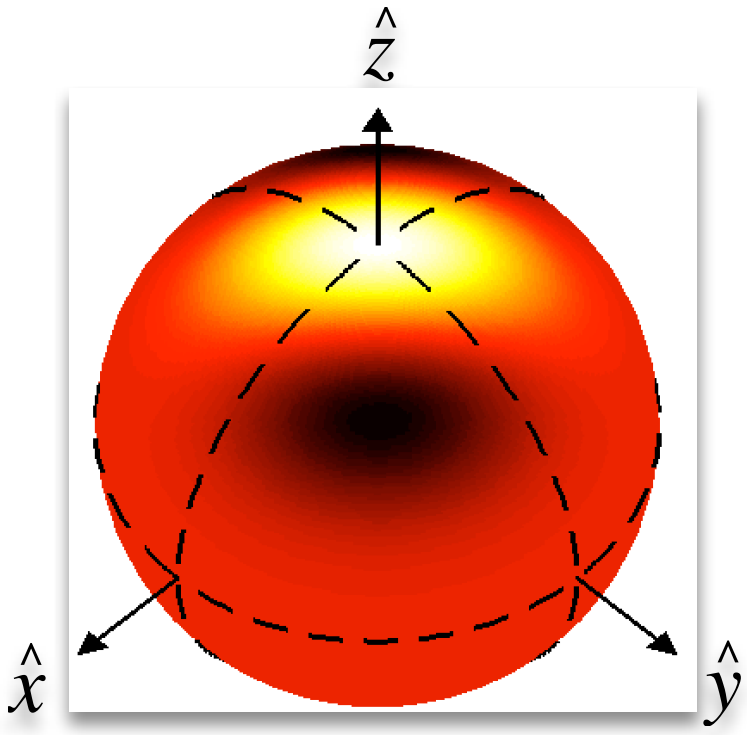


Steerability



Steerability

By linear combination the wavelet is rotated at any continuous angle $\chi \in [0, 2\pi)$



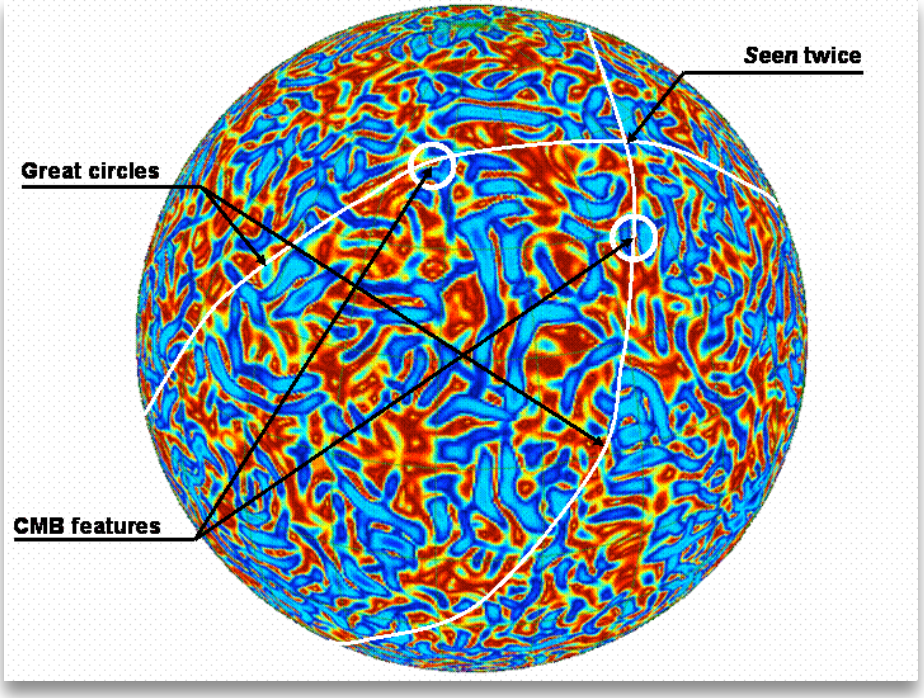
3 basis filters ↔ 3 local morphological measures: orientation, signed-intensity, and elongation!

.II.

Scale-space CMB analyses

Statistical isotropy

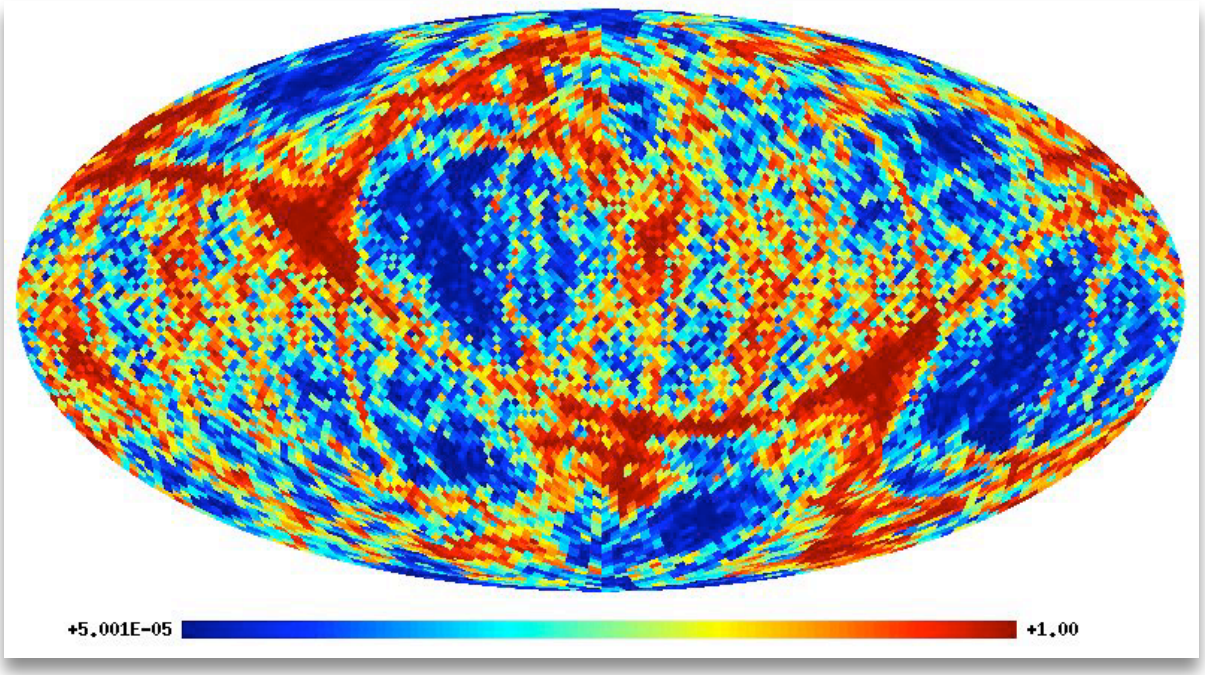
Illustration of the alignment analysis of local CMB features in wavelet space



Signed-intensities at one scale

Statistical isotropy

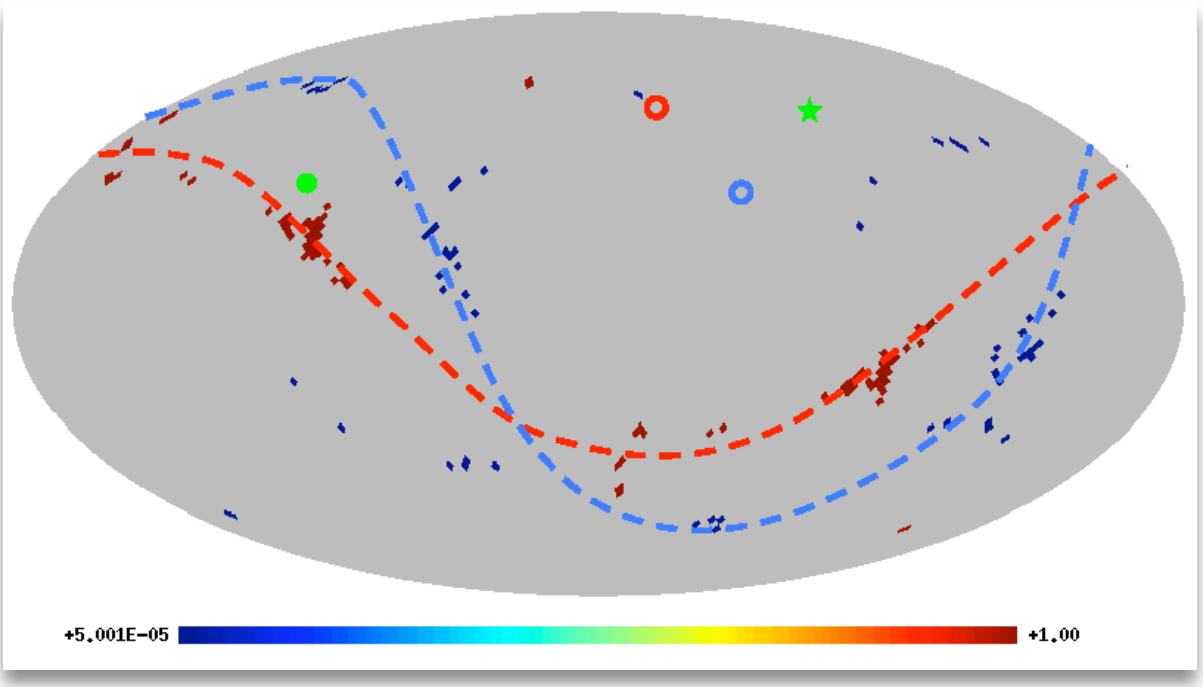
Cumulative probability map of total weights for the alignment of local CMB features in the WMAP3 coadded map



Mollweide projection, Nside = 32, 2GD wavelet scale 8.3°

Statistical isotropy

Total weight **anomalies** (scale 8.3°, S.L. 0.83%) **synthesizing ecliptic poles and CMB dipole axes**

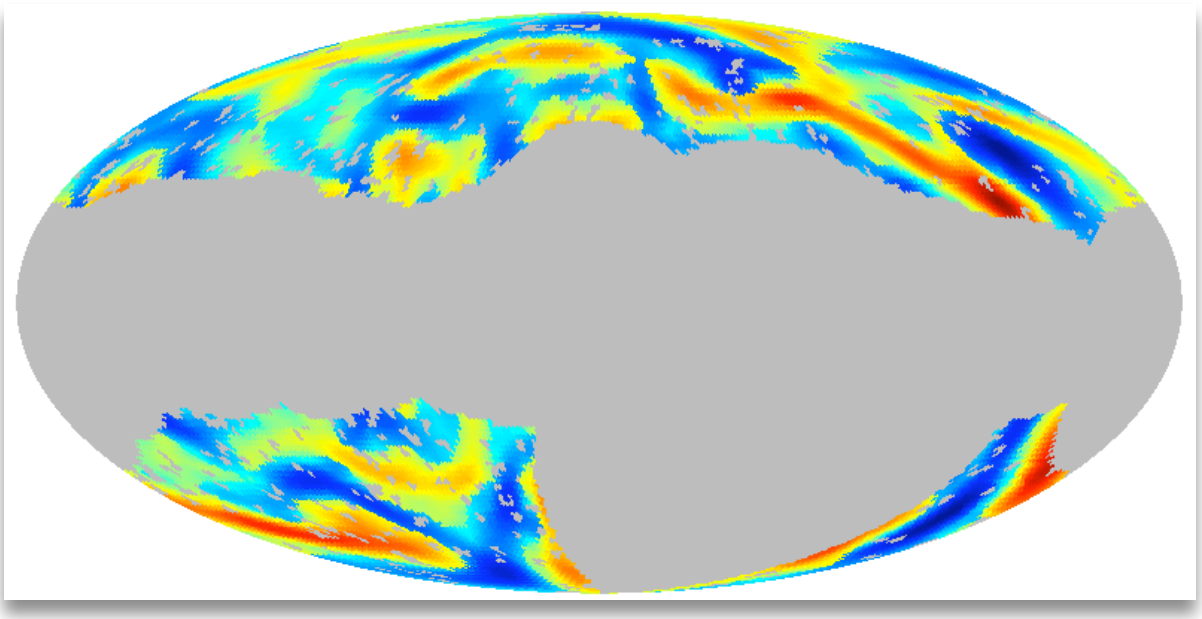


Mollweide projection, Nside = 32, 2GD wavelet scale 8.3°

• *PRL 96 (2006) 151303*

Dark energy

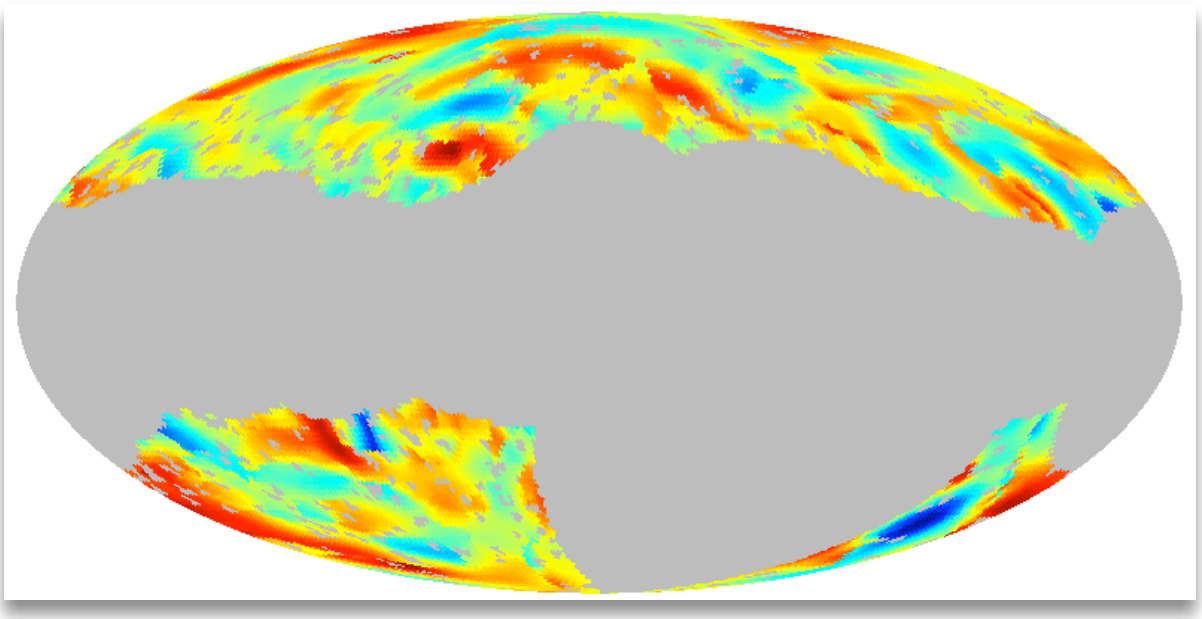
Signed-intensities in the **NVSS galaxy data** (extended mask)



Mollweide projection, Nside = 64, 2GD wavelet scale 13.3°

Dark energy

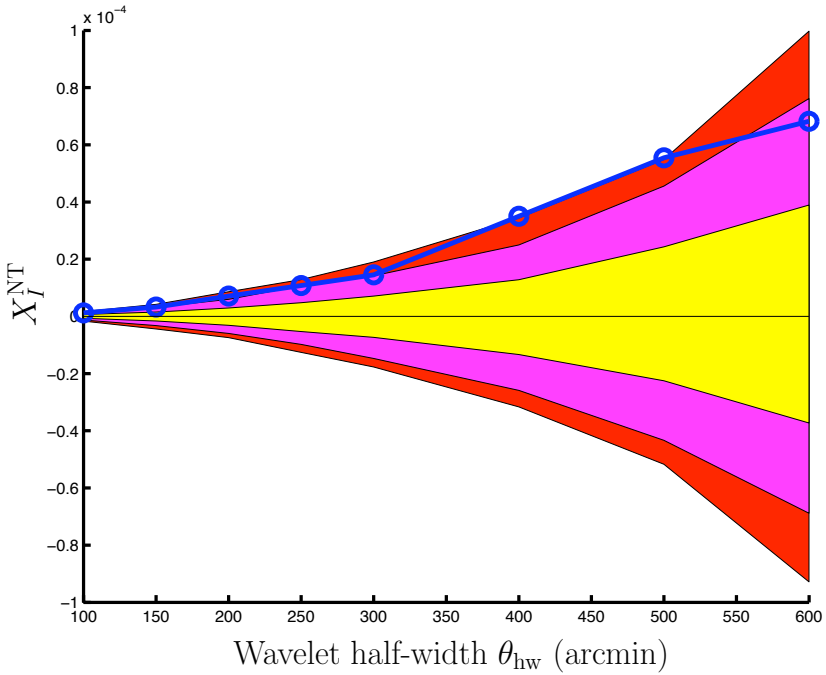
Wavelet coefficients in the WMAP3 coadded map **with orientation matched to NVSS data** (extended mask)



Mollweide projection, Nside = 64, 2GD wavelet scale 13.3°

Dark energy

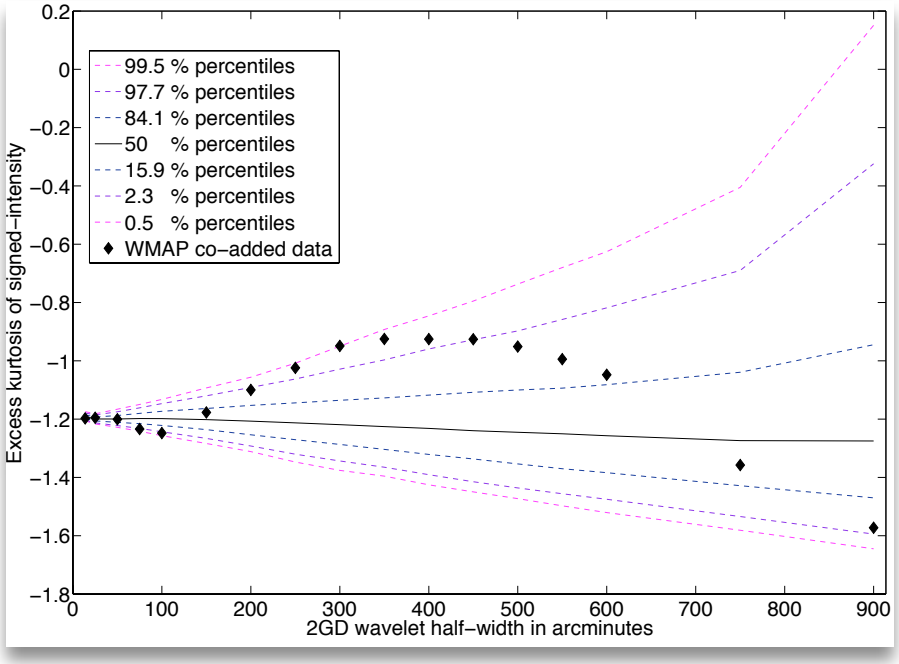
Evidence for dark energy through cross-correlation of WMAP3 and NVSS **matched intensities** (scale 13.3°, global S.L. 0.1 %), **but no detection either in orientation or elongation**



• *MNRAS 384 (2008) 1289*

Gaussianity

Kurtosis anomaly (scale 10°, S.L. 0.5%) **of the signed-intensities** in the WMAP3 coadded map, **but no detection either in orientation or elongation**



• *MNRAS 385 (2008) 939*

.III.

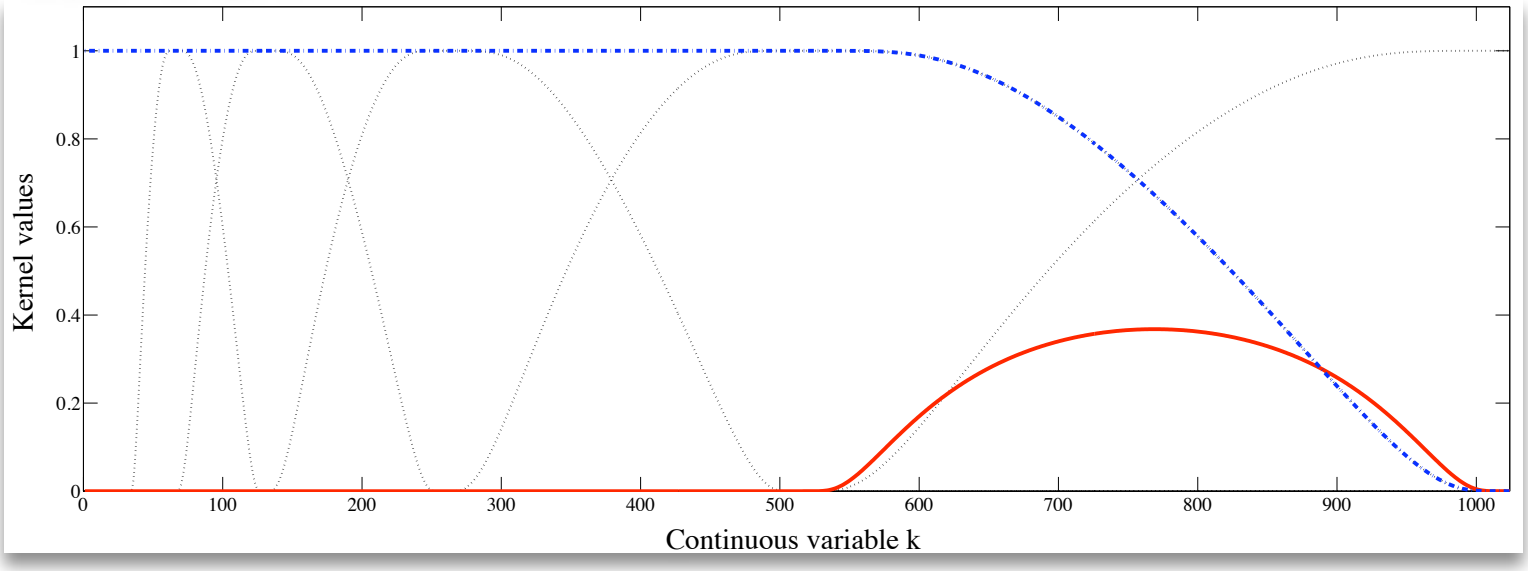
Scale-discretized wavelets on the sphere

Scale-discretized wavelets

Signals are analyzed by local filters which can be translated, rotated, and... **linearly dilated in harmonic space with a discrete dilation factor**

$$\widehat{(\Gamma_{\alpha^j})}_{lm} = \tilde{K}_\Gamma\left(\alpha^j l\right) S^\Gamma_{lm} \qquad |m| < N$$

$$0 \leq j \leq J \leq J_B(\alpha)$$

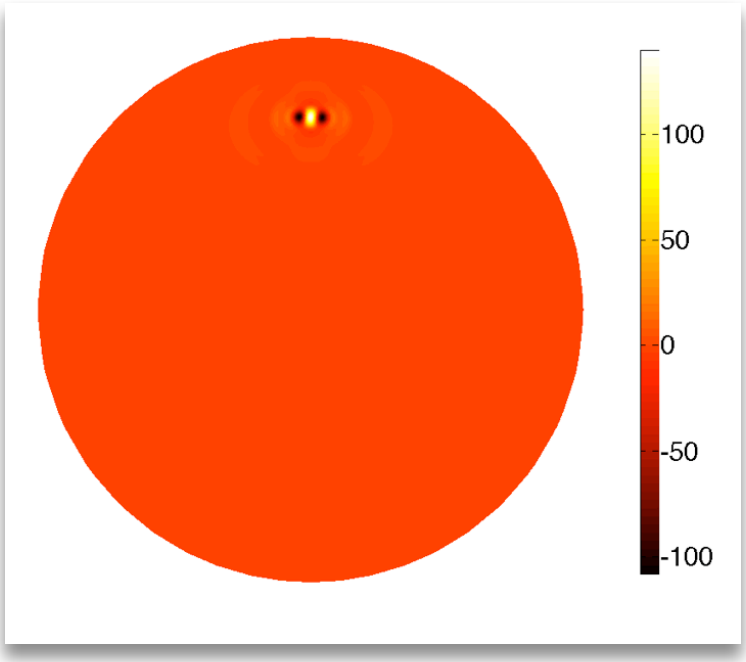


+ perfect reconstruction of signals with the filter bank

- *arXiv:0712.3519v1 [astro-ph]*

Discrete scales

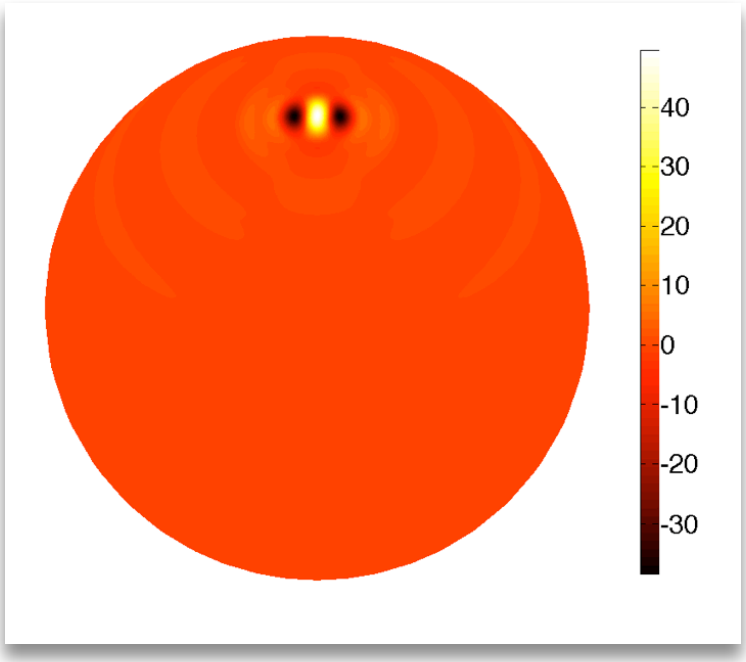
Example wavelet (B=128, N=3, α=2): scale 1



Wavelet frequency range l ∈ (32,128)

Discrete scales

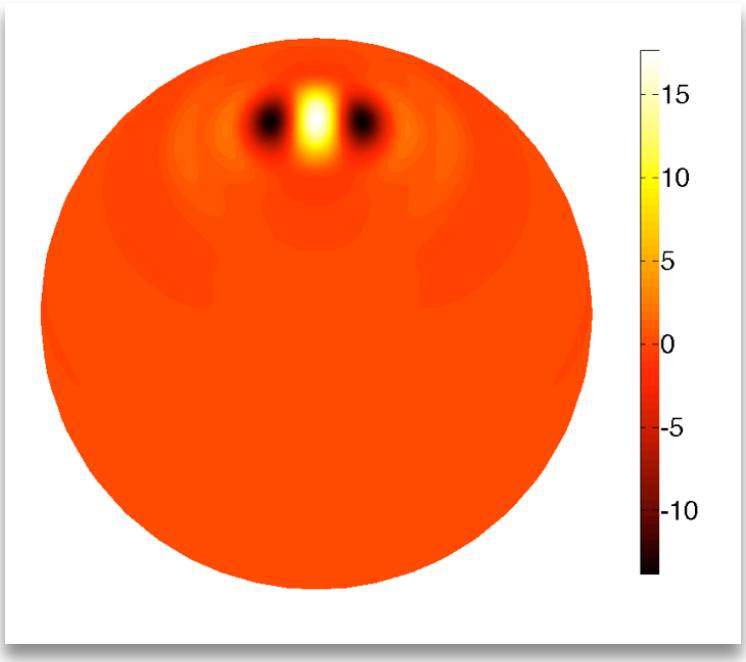
Example wavelet (B=128, N=3, α=2): scale 2



Wavelet frequency range l ∈ (16,64)

Discrete scales

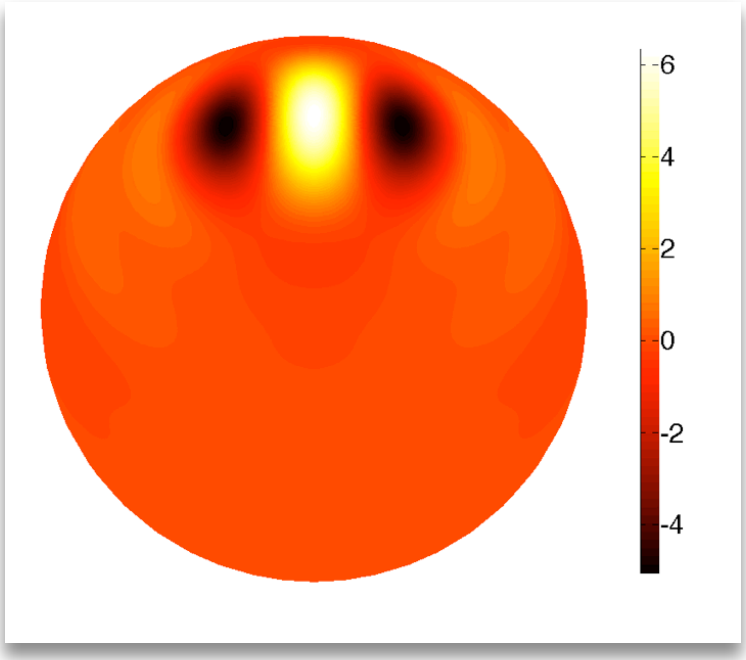
Example wavelet (B=128, N=3, α=2): scale 3



Wavelet frequency range l ∈ (8,32)

Discrete scales

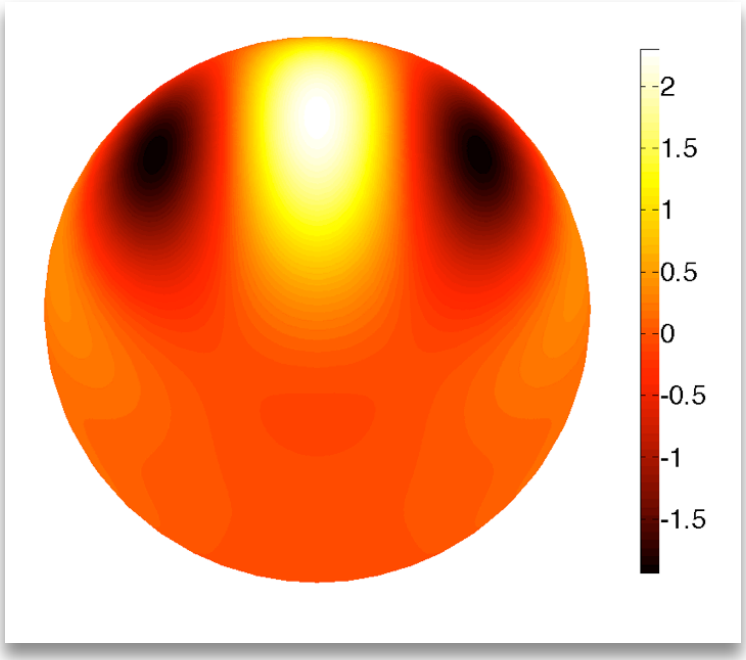
Example wavelet (B=128, N=3, α=2): scale 4



Wavelet frequency range l ∈ (4,16)

Discrete scales

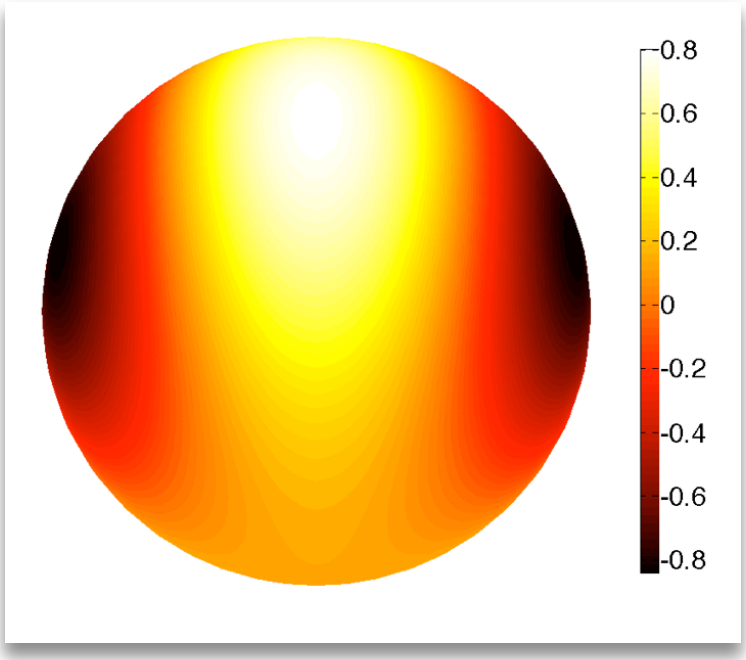
Example wavelet (B=128, N=3, α=2): scale 5



Wavelet frequency range l ∈ (2,8)

Discrete scales

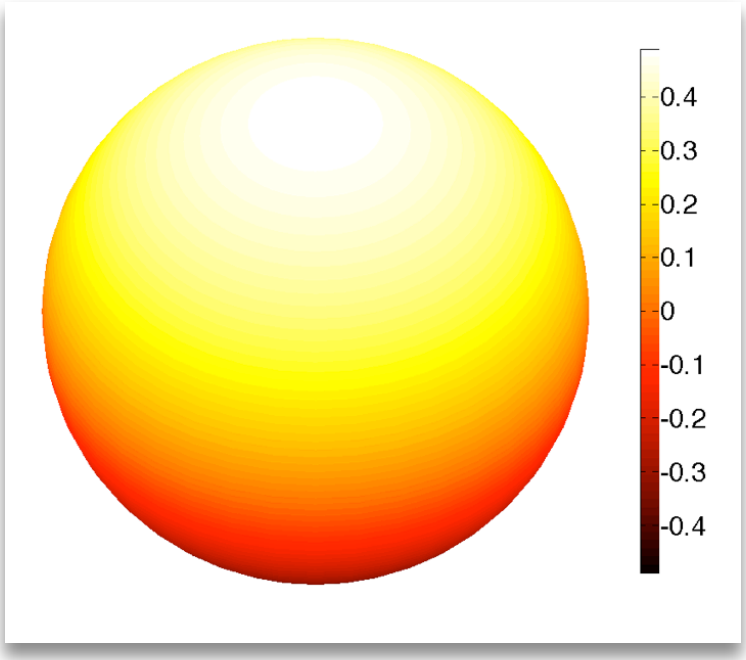
Example wavelet (B=128, N=3, α=2): scale 6



Wavelet frequency range l ∈ (1,4)

Discrete scales

Example wavelet (B=128, N=3, α=2): scale 7



Unique wavelet frequency l = 1

.IV.

Non-Gaussian CMB component denoising

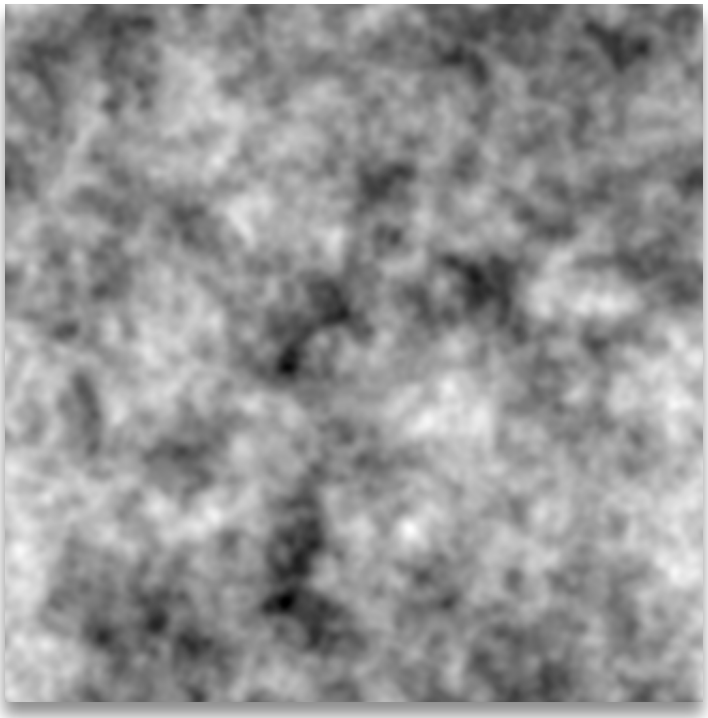
Cosmic strings

Non-Gaussian CMB temperature component induced by cosmic strings: typical temperature steps

FWHM 1', 7.2°x7.2° f.o.v., Fraisse et al.
arXiv:0708.1162v1

Cosmic strings

Gaussian CMB temperature component induced by adiabatic perturbations



FWHM 1', 7.2°x7.2° f.o.v., Fraisse et al.
arXiv:0708.1162v1

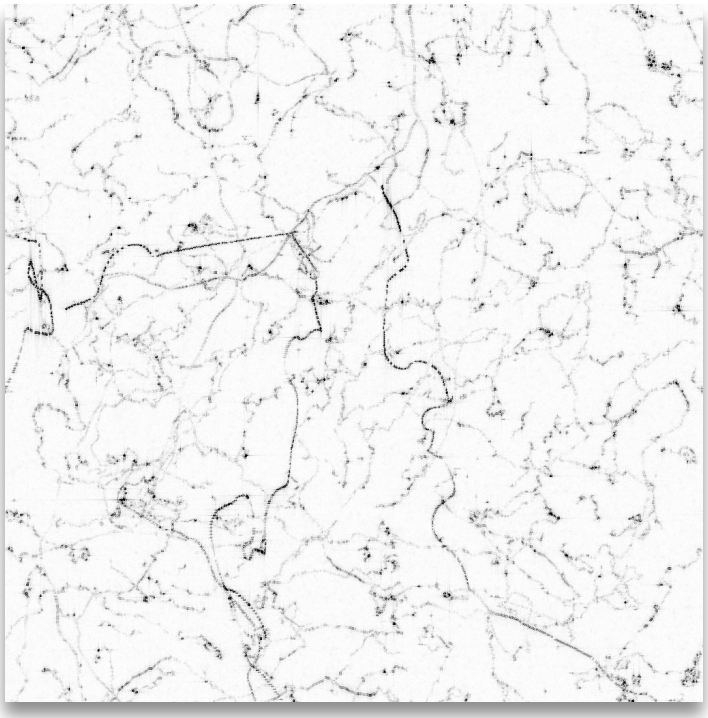
Cosmic strings

Noisy signal gradient

String tension $G\mu = 4 \times 10^{-8}$

Cosmic strings

Signal gradient after statistical denoising with scale-discretized steerable wavelets



String tension $G\mu = 2 \times 10^{-9}$

- *Tech. rep. EPFL-LTS-06.2008*

Conclusion

Whatever the manifold... the plane or the sphere...

Steerability ⟷ Discrete scales

Wavelet formalisms

↓ ↓

Analysis of
local morphological measures

Features identification
and reconstruction

↘ ↙

Efficient
scale-space analysis
of the CMB