Multispectral Analysis based on Wavelet Fusion & Sparse Decomposition

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Virtual Observatory & Multiband Imaging

- Access by ALADIN to a large set of sky surveys
- Consistent detection between images
- Deepest analysis
- Object measurements consistent between bands
- Spectral class identification
- Class mapping

Image Fusion in the pixel space

- Co-addition & related sufficient statistics
  - $x_n = c_n\, x + n_n$ (n_n Gaussian noise s.d. σ_n),
  - $y = \Sigma c_n x_n / \sigma_n^2$ *is a sufficient statistics for the fusion*
  - Best resulting SNR
- What scaling factor c_n ?
  - A global factor on the image → true co-addition
  - Variation with the pixel spectral distribution
- $c_n = x_n$ → Chi-2 images

$$y = \sum \frac{x_n^2}{\sigma_n^2}$$

  - Noisy images
  - Denoising of the Chi-2 images
  - PSF variations → Fusion in a wavelet transform space

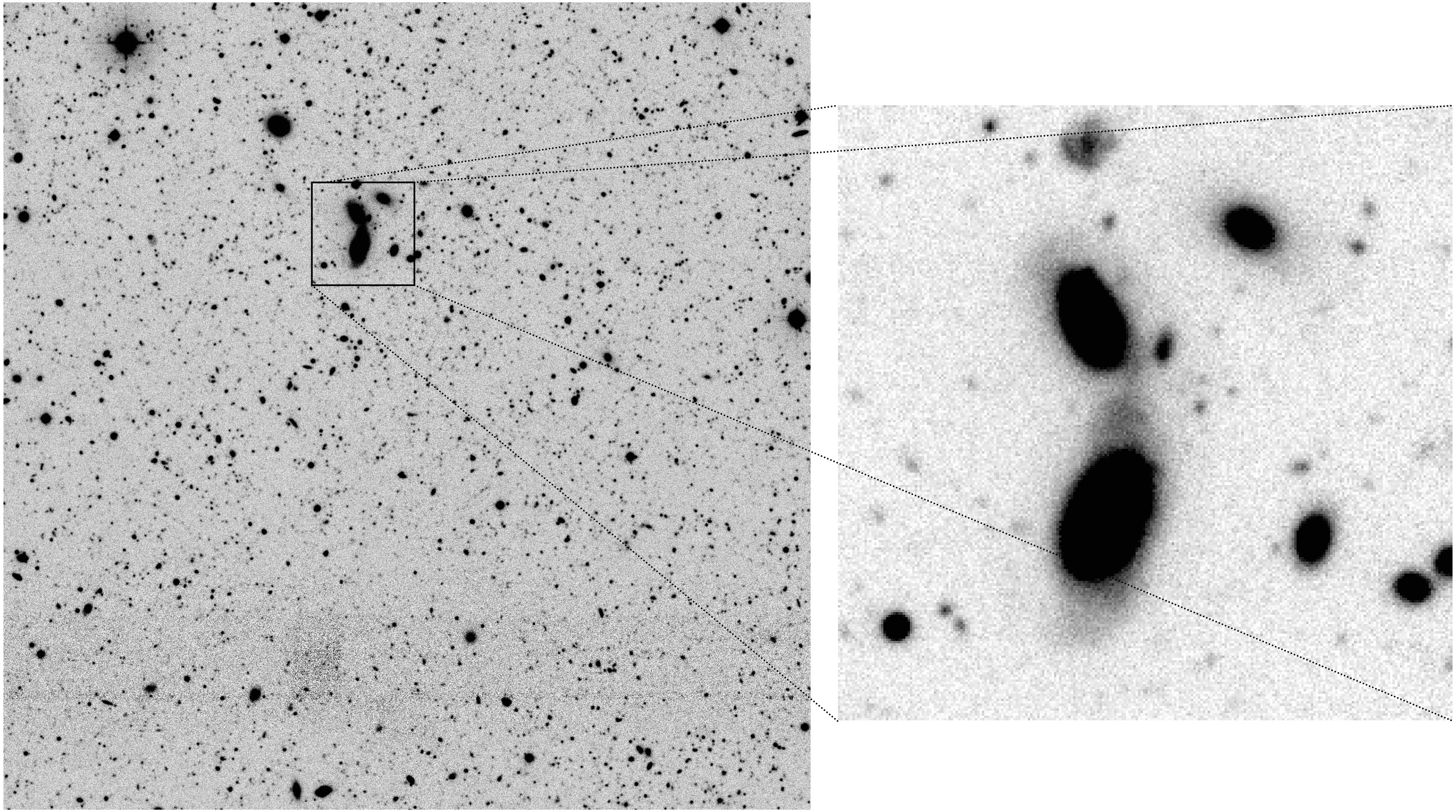
Fusion in the wavelet space

- PSF variation with the color
- Chi-2 fusion scale / scale
  - A wavelet coefficient has a null mean!
  - Associated sign → linear combination

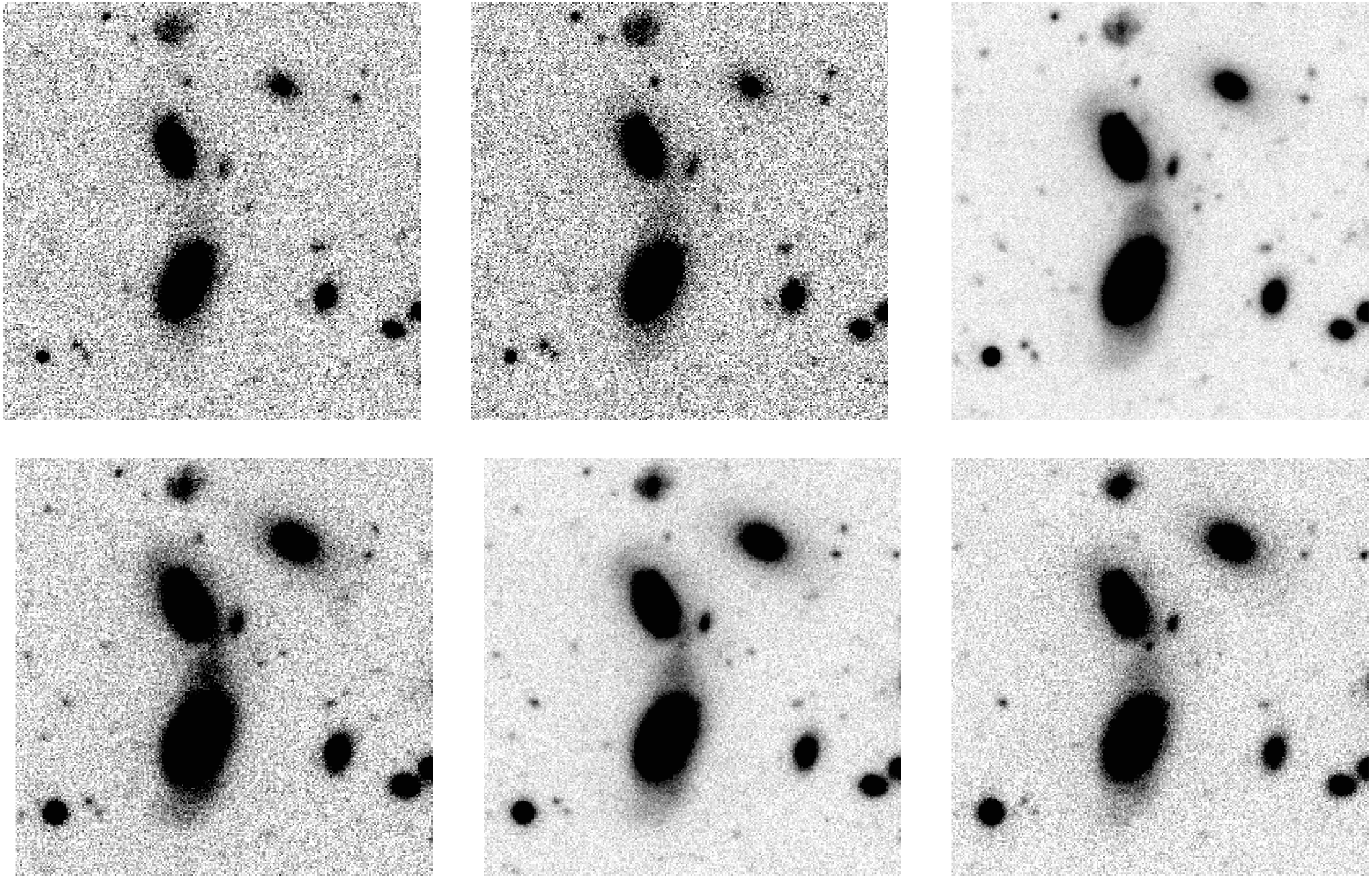
$$z = \sum \frac{x_n}{\sigma_n} \qquad\qquad w = \sqrt{y} \times sign(z)$$

- Applied wavelet transforms
  - À trous algorithm >> too redundant
  - Pyramidal >> Inversion
  - Pyramid of Laplacians
  - DWT

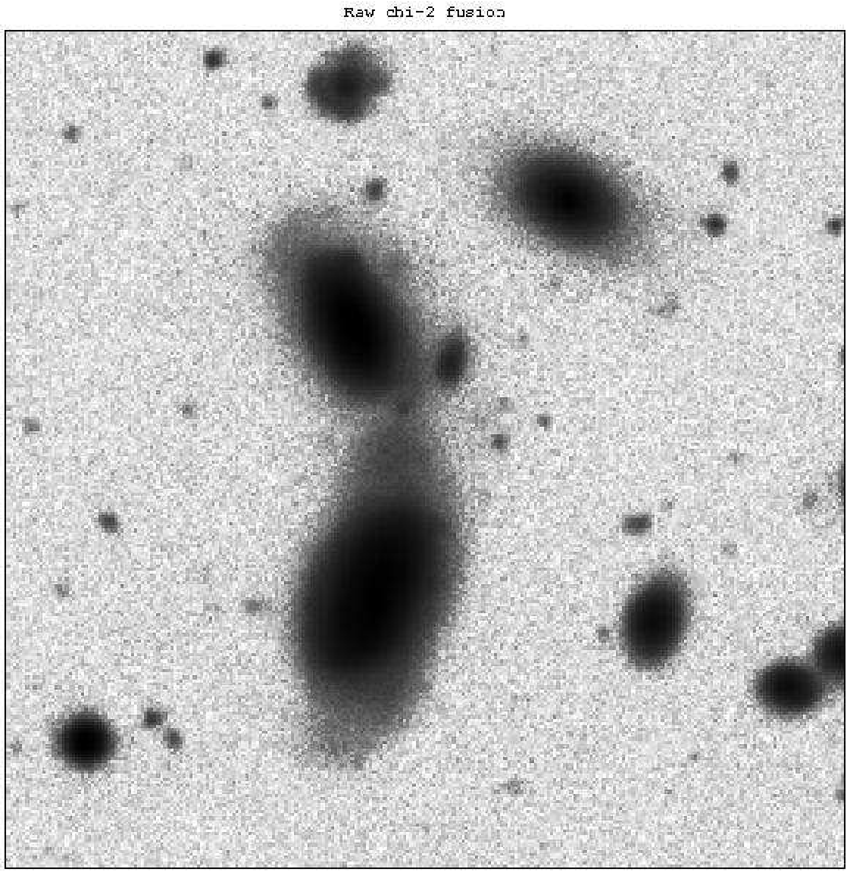
B Image

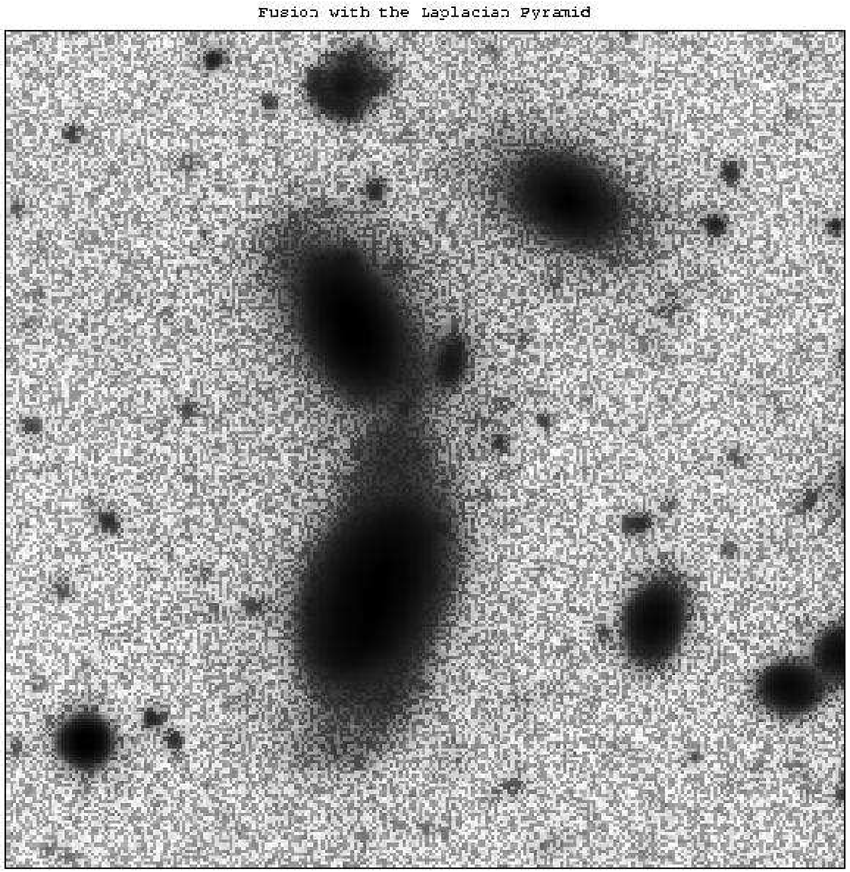


U-Up-B-V-R-I Images



Fusion in the wavelet space





- Thresholding
  - Related statistics

Object Decomposition

- Pixel decomposition (cf. Sextractor)
  - Background mapping
  - Thresholding & field labelling
  - Deblending
  - Field variation with the color
- Multiscale Vision Model
  - Thresholding in WTS
  - Object as local maxima in WTS
  - Multiscale mask & object reconstruction
  - Variation of the multiscale mask with the color

→ *Sparse decomposition common to the images*

Scaling function & sparsity

- The multiresolution block

$$f(i,k) = \frac{1}{2^i}\int_{-\infty}^{+\infty} f(x)\varphi(\frac{x-k}{2^i})dx$$

- Dilation equation

$$\frac{1}{2}\varphi(\frac{x}{2}) = \sum h(n)\varphi(x-n)$$

Information at scale i can be merged into a more significant coefficient at scale $i+1$

- Search of a sparse decomposition
  - Variational approach (ex. basis pursuit)
  - Greedy algorithm (ex. matching pursuit)

MP with scalets (1)

The à trous algorithm:

$$f(i+1,k)=\sum h(n)\,f(i,k+2^i n)$$

- That we can write as:

$$f(i+1,k)=\sum h(i,m)\,f(k+m)$$

- With: $$h(i,m)=\sum_n h(i-1,m-2^i n)h(n)$$
- Scalet at (i,k_0) $$f(k_0+m)=a(i,k_0)h(i,m)$$
- We get:

$$a(i,k_0)=\frac{\sum f(k_0+m)h(i,m)}{\sum h^2(i,m)}$$

MP with scalets (2)

- l_2 reduction

$$n(i,k_0) = \frac{[\sum f(k_0+m)h(i,m)]^2}{\sum h^2(i,m)} = (SNR)^2$$

- Greedy algorithm with scalet patterns $h(i,m)$
- That does not work
  - Too slow algorithm (undecimated/pyramidal)
  - Difficulty to parallelize
  - Non orthogonality → High pattern coupling

Local backgound removal

- The background is a spurious component
- Local estimation

$$f(k) = a(i,k_0)\,p(i,m) + b(i,k_0)$$

- Adjustment window

$$R(i,k_0) = \sum_m \varpi(i,m)\left[f(k) - a(i,k_0)\,p(i,m) - b(i,k_0)\right]^2$$

- New cross-product

$$a(i,k_0) = \sum f(k_0 + m)\,r(i,m)$$

- With

$$r(i,m) = \frac{S_0\,p(i,m)\,\omega(i,m) - S_1\varpi(i,m)}{S_0 S_2 - S_1{}^2}$$

Connection to the à trous WT

- ℓ_2 reduction is not the available criterion

→ *coefficient SNR*

- $p(i,m)$ pattern & fast algorithm
- Fit with a 2^i step : $m=n\,2^i$
- We set

$$\varpi(i,m) = h(n) \qquad\qquad p(i,m) = \delta(n)$$

- This leads to the wavelet coefficient

$$a(i,k_0) = \frac{1}{1-h(0)}[f(i,k_0) - f(i+1,k_0)]$$

The Pyrels

- Les $h(i,m)$ are generated by the à trous algorithm → we call them *pyrels* (pyramid elements).
- They are *scalet type* pyrels
- Other types can be generated by a modification of the local fit
- Experimentally : binomial filter
  - The $h(i,m)$ series tend to the B-splines
  - Compacity, regularity → quite Gaussian

MP the AT WT

1. Wavelet transform
2. Suprema identification
3. *Threshold =* α *Abs(Max)* → Coupling
4. Amplitude adjustment (coupling reduction)
5. Image reconstruction
6. Wavelet transform & subtraction
7. Test on the residuals
8. Come back to step 2
9. Global background estimation and adding

The Pyramidal Algorithm

- The Pyramidal wavelet transform
- Same pyrel pattern $h(i,m)$
- Suprema identification on a pyramid
- Reduction of the coupling between the pyrels
  - Increase the algorithm stability
- Background reduced to a constant
- Can work on only positive pyrels

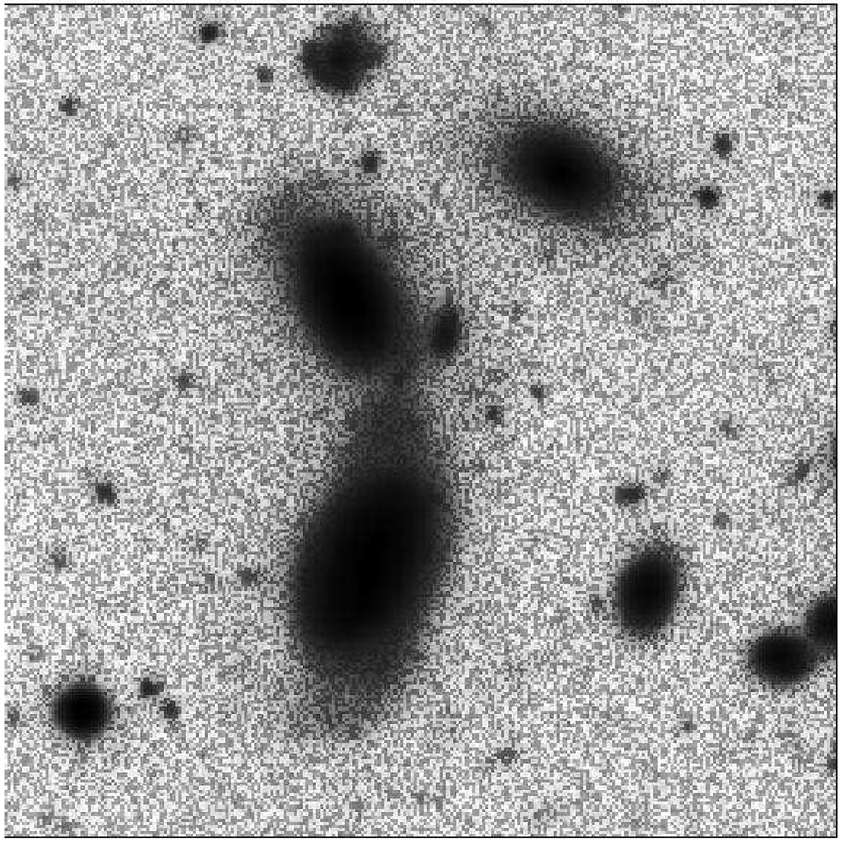
Two-Dimensional Algorithms

- What scalet ?
  - Variable separation for the scaling function
  - Quasi isotropic analysis with B-spline
- What wavelet ?
  - A trous / Pyramidal
  - Difference between approximations
  - *Oriented patterns → other pyrel types*
- Transposition of the 1D algorithms
- Possibility of positive decomposition

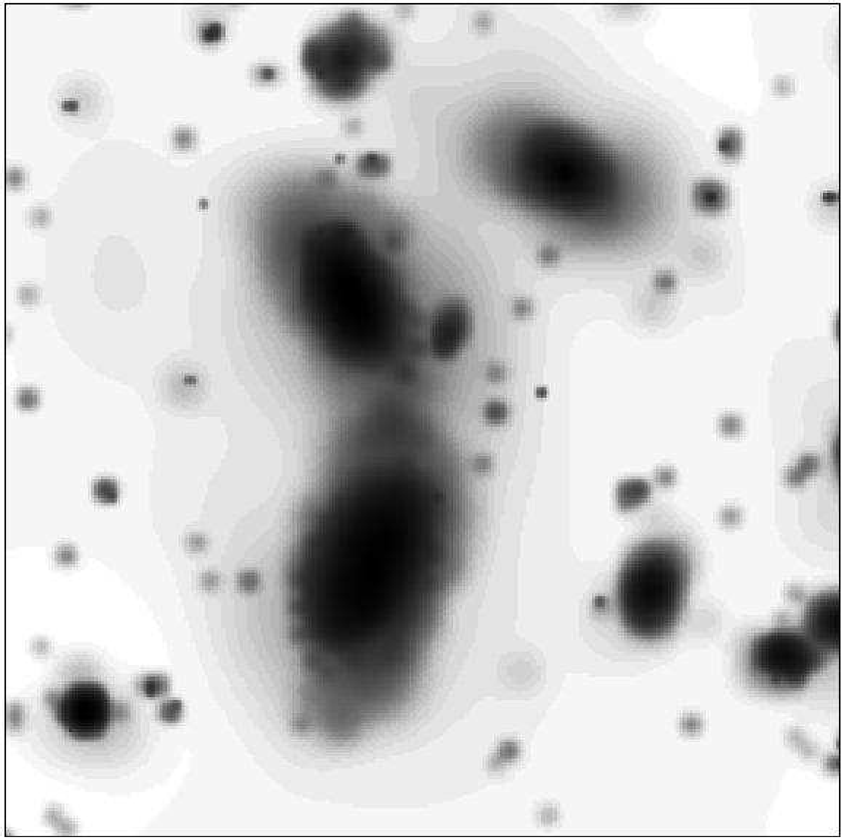
MSMPPy2 on the Fusion Image

- Significant wavelet coefficients : 2% pixels
- Pyrels : 0.43 %

Fusion with the Laplacian Pyramid

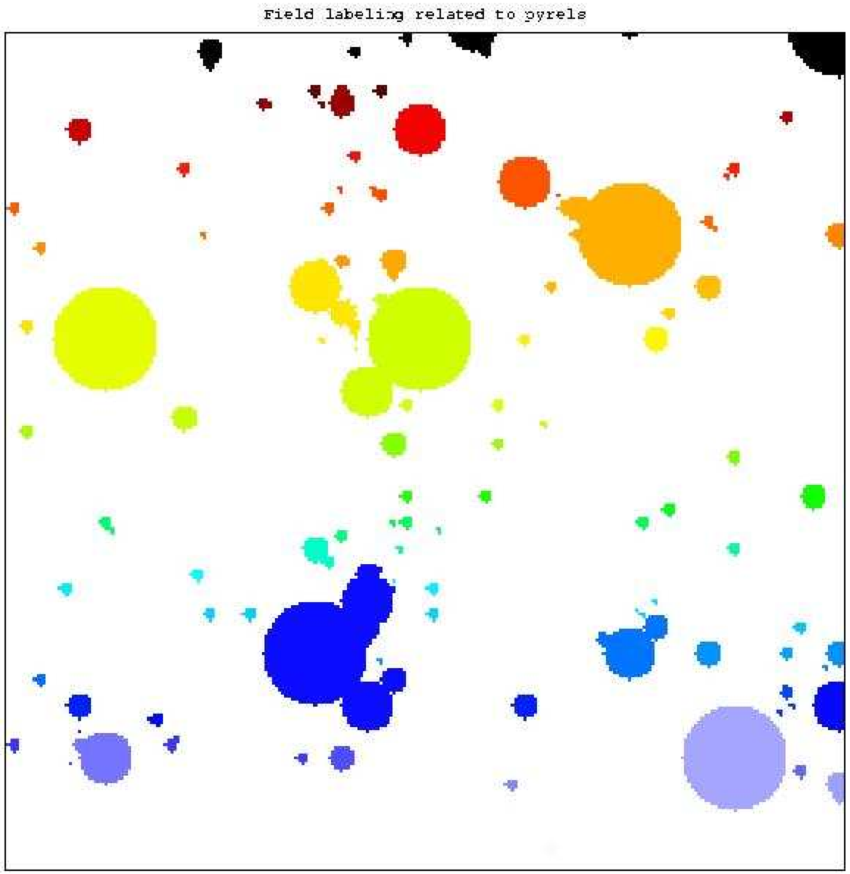


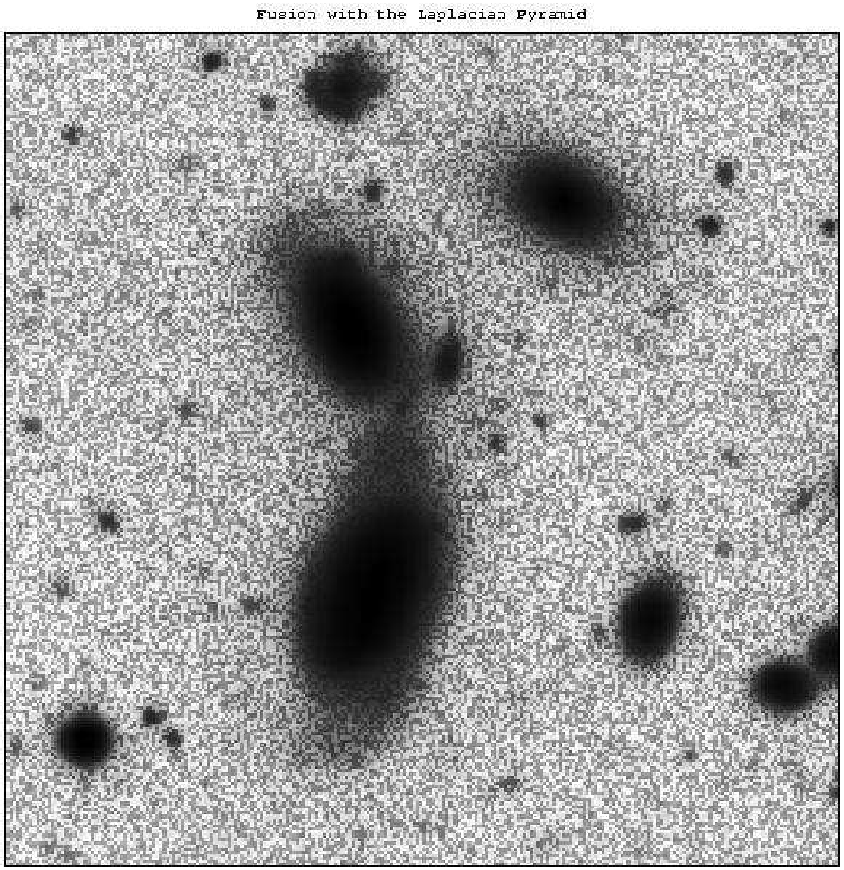
Pyrel decomposition of the fusion image



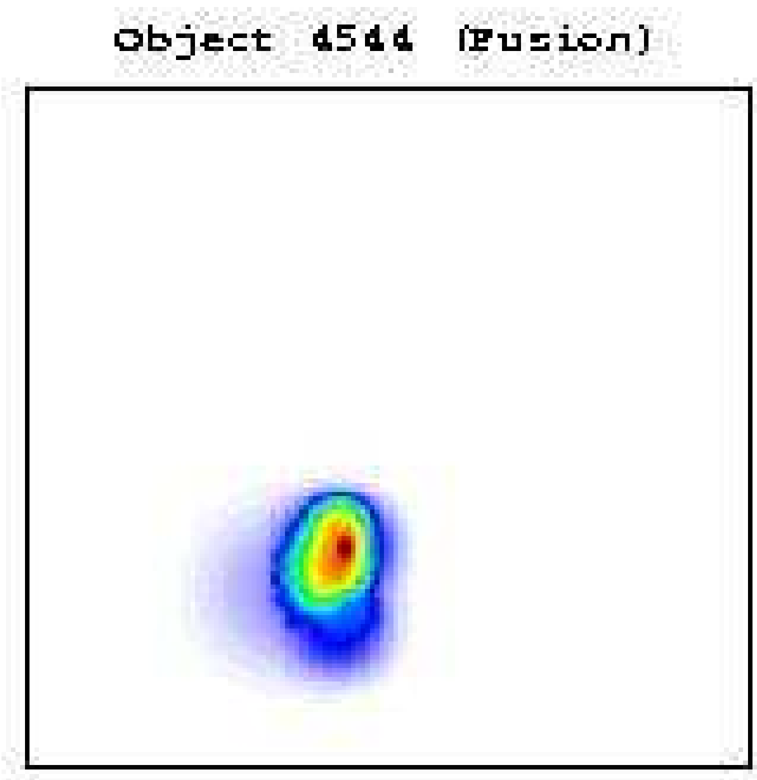
Multiscale Field labelling

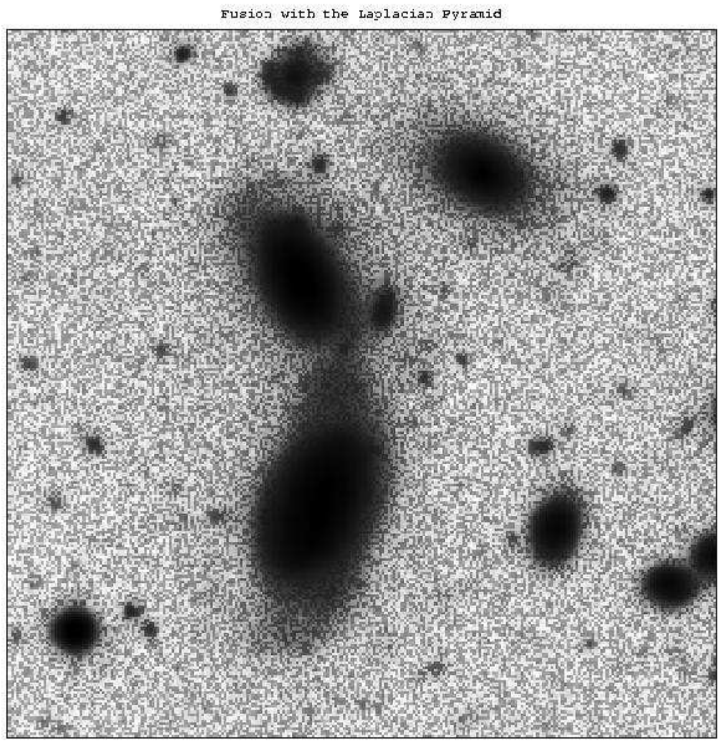
- The pyrels from which the centers are less than a radius depending of their scale have the same label





Object Reconstruction

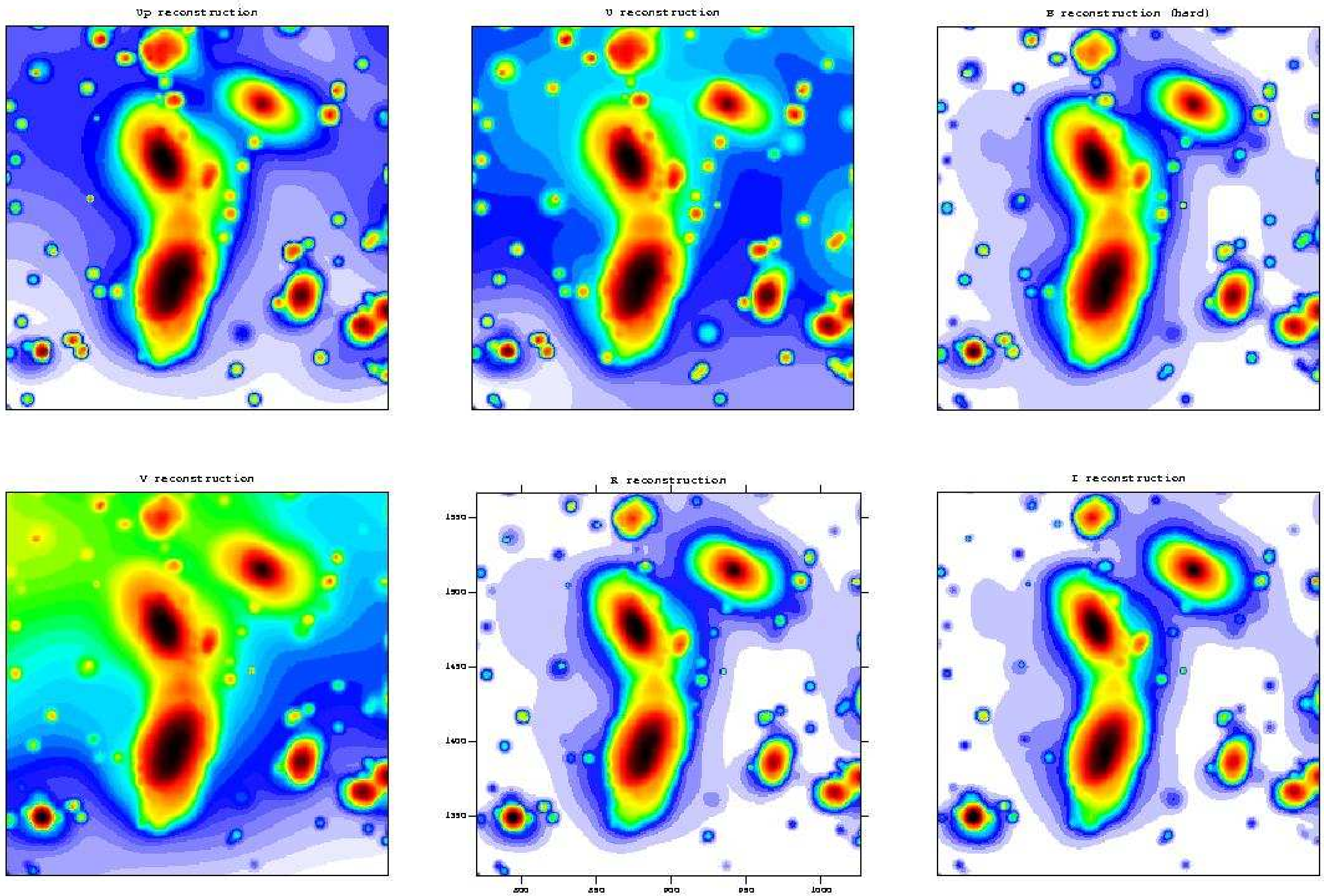




- After the field labelling it easy to associate to merge pyrels into objects
- The image of each object can be restored from the pyrels having its label

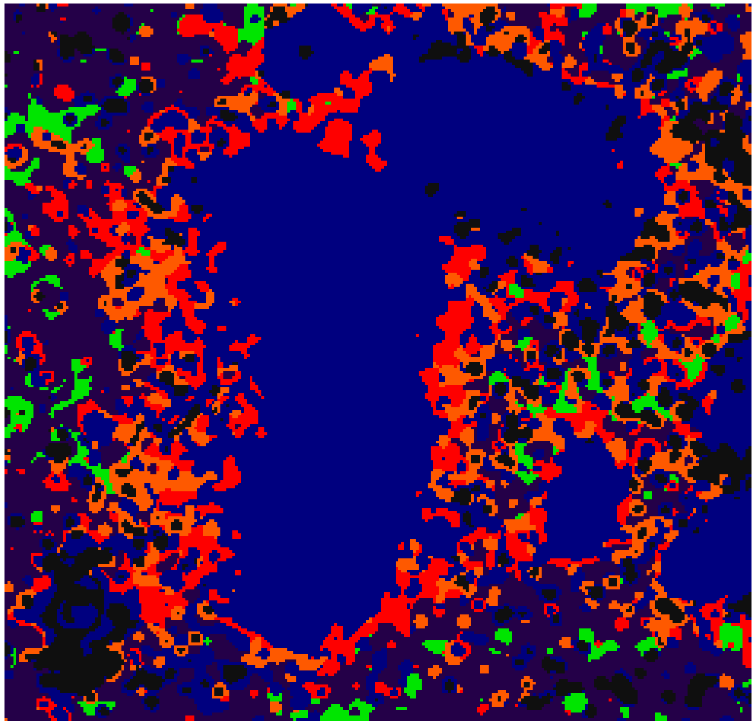
Restoration from the Pyrel Map

- The pyrel decomposition carries out a set of functions which can be used for restoring each image
- Only the pyrel amplitudes are fitted from their related wavelet coefficients
- That concentrates information on few coefficients leading to clean images
- Thus consistent measurements between colors can be derived


Up reconstruction
U reconstruction
B reconstruction (hard)
V reconstruction
R reconstruction
I reconstruction

Spectral Analysis

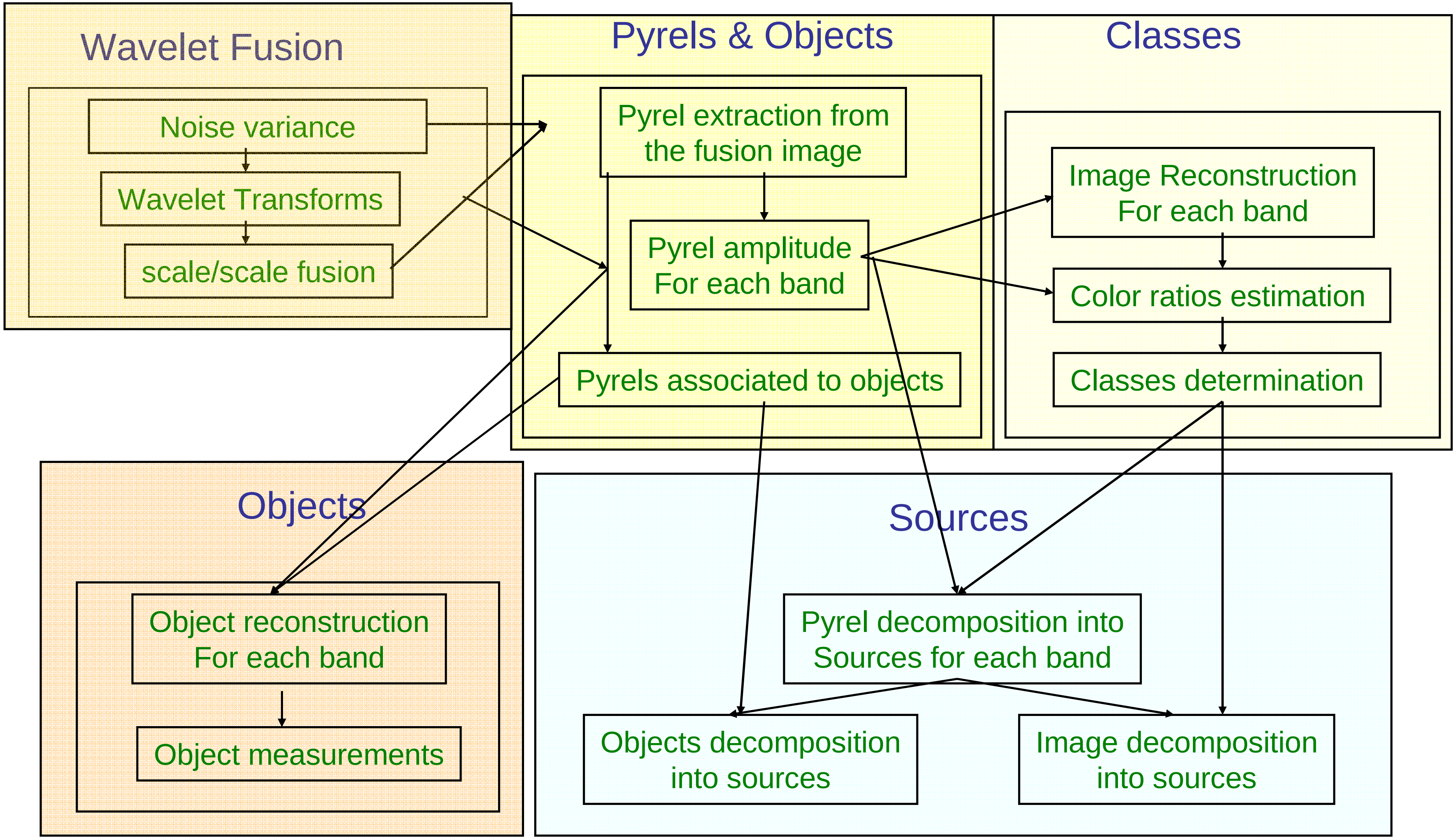
- Classical color indices
  - Negative / null fluxes
  - Noisy indices
  - Non additivity from components
- Use of ratios / weighted mean
  - Ratio noise / weighted mean
  - Additivity from components
- Classification from the ratios
  - Weighted K-means method
  - ~10 classes
  - Pixels /Pyrels
  - Insufficient class separation



Source Decomposition

- Object : pixel / pyrel values as a mixture of spectral energy distributions (SED)
- Matching pursuit with SED
- Source non orthogonality
- Source images can be determined from the pyrel decomposition

General Organisation



Conclusion

- Work in progress
  - Optimization by a variational approach
  - Oriented pyrels
  - New pyrel labelling for object decomposition
  - Bayesian coefficient softening
  - Fusion during the pyrel identification process
  - Taking into account the PSF
- Can be applied to large images
- Is information lost after the fusion?
  - Pyrel decomposition and Blind source separation